PROGRESS IN EVALUATING A POSSIBLE ELECTROMAGNETIC INTERACTION ENERGY IN A GRAVITATIONAL FIELD

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Lorentz-Poincaré version of special relativity

Special relativity (SR) is usually thought to enforce us to abandon the classical concepts of separated space & time.

But this is not true in the Lorentz-Poincaré version of SR, which sees the space contraction and time dilation as absolute effects of motion through the "ether"
(Prokhovnik: The Logic of Special Relativity, C.U.P. (1967). + others)

The inertial time of the preferred inertial frame (ether) is thought of as the "true time", and the simultaneity defined in the ether is thought of as the "absolute simultaneity".

Yet because Lorentz-Poincaré theory is really equivalent to SR, that ether is indetectable and all inertial frames are equally good candidates for being the ether!

Scalar ether theory of gravitation (SET)

But SR doesn't include gravity. Gravity might violate relativity and reveal the ether. (Note that even GR doesn't obey SR's relativity principle, since there are no global inertial frames and no global Lorentz transformations in GR.)

Such a violation does happen in SET, which starts from gravity as being Archimedes' thrust in a fluid ether filling the space (MA, Found. Phys. **34**, 1703 (2004)).

SET coincides with SR when the gravitational field vanishes.

It endows spacetime with two metrics: a flat "background" metric γ^0 and a curved "physical" metric γ. Motion is defined by an extension of Newton's 2nd law to a curved spacetime.

ELECTRODYNAMICS IN THE PRESENCE OF GRAVITY

The eqs. of electrodynamics of GR rewrite those of SR by using the "comma goes to semicolon" rule: $_{,\nu} \rightarrow {}_{;\nu}$ (partial derivatives → covariant derivatives).

Not possible in SET, for the Dynamical Equation isn't generally $T^{\lambda\nu}_{\ ;\nu} = 0$ (which rewrites $T^{\lambda\nu}_{\ ,\nu} = 0$ valid in SR).

Electrodynamics in the presence of gravity in SET

In SET, first Maxwell group unchanged. Second group was got by applying the Dynamical Eqn of SET to a charged medium in the presence of Lorentz force, assuming that (as is the case in GR):

(A) Total energy tensor $\boldsymbol{T} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}}$.

The additivity (A) leads to a form of Maxwell's 2nd group in SET (MA, Open Phys. **14**, 395 (2016), or Proc. IARD 2016: J. Phys. Conf. Ser. **845**, 012014 (2017)).

But that form of Maxwell's 2nd group in SET predicts charge production/destruction at untenable rates ⇒ *discarded* (MA, Open Phys. **15**, 877 (2017)).

Necessity of the interaction tensor in SET

The additivity assumption (A) is contingent and may be abandoned.

Means introducing "interaction" energy tensor $\boldsymbol{T}_{\text{inter}}$ such that

$$\boldsymbol{T}_{\text{(total)}} = \boldsymbol{T}_{\text{charged medium}} + \boldsymbol{T}_{\text{field}} \underline{+\boldsymbol{T}_{\text{inter}}}\,. \tag{1}$$

One then has to constrain the form of $\boldsymbol{T}_{\text{inter}}$ and derive eqs for it.

Form of the interaction tensor

In SR, the additivity assumption (A) holds, thus $\boldsymbol{T}_{\text{inter}} = 0$.

In SET we may impose that $\boldsymbol{T}_{\text{inter}}$ should be Lorentz-invariant in the situation of SR, i.e. when the metric γ is Minkowski's metric γ^0 $(\gamma^0_{\mu\nu} = \eta_{\mu\nu}$ in Cartesian coordinates).

This leads uniquely to the following definition:

$$T^{\mu}_{\text{inter}\ \ \nu} := p\, \delta^{\mu}_{\nu}, \qquad \text{or} \quad (T_{\text{inter}})^{\mu\nu} := p\, \gamma^{\mu\nu}, \tag{2}$$

with some scalar field p. (MA, J. Geom. Sym. Phys. **50**, 1–10 (2018); MA, Open Phys. **16**, 488 (2018))

INTERACTION ENERGY

Corresponding interaction energy: $E_{\text{inter}} := T^{00}_{\text{inter}} = p\gamma^{00}$.

The medium with energy tensor $(T_{\text{inter}})^{\mu\nu} := p\,\gamma^{\mu\nu}$ can be counted as “dark matter”, because:

- it isn’t localized inside usual matter: $p \neq 0$ at a generic point;
- it’s gravitationally active: $T^{00} =$ source of grav. field in SET;
- it is not usual matter (e.g. no velocity can be defined).

EQUATION FOR THE SCALAR FIELD p (1)

With the interaction energy tensor (2) we have just one unknown more: the scalar field p. So we need just one scalar eqn more.

We may add *charge conservation* as the new scalar eqn. Then the system of eqs of electrodynamics of SET is again closed, and satisfies charge conservation.

Based on that closed system, eqs. were derived that, *in principle,* determine the field p in a given general EM field $(\mathbf{E}, \mathbf{B})$ and in a given weak gravitational field with Newtonian potential U (MA, Open Phys. **16**, 488 (2018)).

EQUATION FOR THE SCALAR FIELD p (2)

Main equation is the following PDE for p:

$$\boxed{\mathrm{div}_4\,(\boldsymbol{G}.\nabla_4 p) := \left(G^{\mu\nu}\, p_{,\nu}\right)_{,\mu} = f.} \tag{3}$$

$G^{\mu\nu}$: the components of antisymmetric spacetime tensor $\boldsymbol{G}$: inverse tensor of EM field tensor of the first approximation, that obeys the flat-spacetime Maxwell equations.

In addition, in Eq. (3), we have

$$\boxed{f := \left(d^i \partial_T U\right)_{,i}.} \tag{4}$$

d^i $(i = 1, 2, 3)$: components of a spatial vector d made with (E, B).

Time derivative $\partial_T U$ taken in the preferred frame.

Homogenizing the PDE for T_{inter}

Need to integrate *on a galactic scale* $r \sim 10^{19}\text{m}$ the PDE (3) for the scalar field p.

But: given fields $\boldsymbol{G}$ and f in (3) vary on scale $r \sim \lambda \simeq 10^{-6}\text{m}$ and $t \sim \lambda/c$, like E and B. No chance to succeed in the integration!

Situation typical of *homogenization theory*.

Aim of that theory: to get "homogenized" PDEs allowing one to describe at the macroscopic scale the medium, assumed periodic or quasi-periodic at a microscopic scale.

For Eq. (3), the "medium" is characterized by the pair of given heterogeneous fields $(\boldsymbol{G}, f)$.

A BIT MORE ON HOMOGENIZATION THEORY

Considers two spacetime variables related by a small parameter $\epsilon \ll 1$:

- slow variable X: browses medium at macroscopic scale
- quick variable, $\mathrm{Y} = \mathrm{X}/\epsilon$: an $O(1)$ variation of it browses the quasi-period of the medium.

Fields are stated to be functions of X *and* $\frac{\mathrm{X}}{\epsilon}$, *periodic* or quasi-periodic w.r.t. $\frac{\mathrm{X}}{\epsilon}$.

Asymptotic expansions are stated, e.g.

$$p^{\epsilon}(\mathrm{X}) = p_0\,(\mathrm{X},\mathrm{Y})\,\epsilon^0 + p_1\,(\mathrm{X},\mathrm{Y})\,\epsilon + O(\epsilon^2), \qquad \mathrm{Y} = \frac{\mathrm{X}}{\epsilon} \quad (5)$$

Homogenizing the PDE for T_{inter}: the different possible ways

Depending on which spacetime variable is considered primordial, three possibilities:

- Time Homogenization: Homogenizat^{n} theory applies quite straightforwardly (IARD 2022). But the remaining space dependence at scale $r \sim \lambda \simeq 10^{-6}$m prevents integration of the PDE at galactic scale.
- Space Homogenization: Homogeniz^{n} theory applies less well. Anyway, time dependence at scale $t \sim \lambda/c$ also prevents integration of the PDE at galactic scale.
- Spacetime Homogenization.

Spacetime homogenization

The PDE (3) for p has just the same form as the *stationary heat conduction equation for the temperature* θ, except that here we have 4-d *spacetime* instead of 3-d space ⇒ May adapt known results: Caillerie, summer school Quiberon 2012.

Main result: homogenized PDE has same form as (3), replacing $\boldsymbol{G}$ by a "homogenized" tensor $\boldsymbol{G}^{\mathrm{H}}$. However, $\boldsymbol{G}^{\mathrm{H}}$ is *not* the local spacetime average of "microscopic" tensor $\boldsymbol{G}$:

$\boldsymbol{G}^{\mathrm{H}}$ obtained by solving a boundary value problem on a local microscopic cell for the linear first-order PDE

$$k^{\mu}\,\chi^{\nu}{}_{,\,\mu} = -k^{\nu} \quad (\nu = 0, ..., 3), \qquad k^{\nu} := G^{\mu\nu}{}_{,\,\mu}. \tag{6}$$

To be solved by finite element method. This has been numerically implemented! But then, still need to solve $\left(G^{\mathrm{H}\ \mu\nu}\,P_{,\nu}\right)_{,\mu} = F$ for the unknown $P := \langle p \rangle$ and with data $F := \langle f \rangle$...

Spacetime averaging

Because $G^{\mu\nu} = -G^{\nu\mu}$, Eq. (3) rewrites as a *first-order* PDE:

$$\boxed{k^\nu p_{,\nu} = f,} \qquad \text{or} \quad \mathrm{k}.\nabla p = f \qquad (\nabla := \nabla_4). \tag{7}$$

And again because $G^{\mu\nu} = -G^{\nu\mu}$, we have

$$\operatorname{div}\mathrm{k} := \operatorname{div}_4\mathrm{k} := k^\nu_{\ ,\nu} := G^{\mu\nu}_{\ \ ,\mu\,,\nu} = 0. \tag{8}$$

Recall: all fields here vary with pseudo-periods $\lambda \simeq 1\mu\mathrm{m}$ and $T \simeq \lambda/c$, *extremely* small w.r.t. galactic scales.

We assume that the fields k and ∇p are "locally macro-homogeneous". I.e., they are *slow variations* of *macro-homogeneous fields*. In brief, the latter means that the averaged fields $\mathrm{K} = \overline{\mathrm{k}}$ and $\overline{\nabla p}$ are constant. Details follow.

MACRO-HOMOGENEOUS FIELDS k AND ∇p

We provisionally forget the slow variation of the fields. We assume (MA, Arch. Mech. **43**, 191 (1991) for k = stress & p = velocity fields):

1) $\mathsf{k} = \mathsf{k}_0 + \delta\mathsf{k}$ with $\delta\mathsf{k}$ bounded and, for cubes Ω of side $R(\Omega)$,

$$\frac{1}{V(\Omega)} \int_\Omega \delta\mathsf{k} \, \mathrm{d}V \to 0 \quad \text{as} \quad R(\Omega) \to \infty. \tag{9}$$

2) $p(\mathrm{X}) = \mathrm{g}_0.\mathrm{X} + \delta p$, with ($\partial\Omega$ being the boundary of the cube Ω):

$$\frac{1}{V(\Omega)} \int_{\partial\Omega} |\delta p| \, \mathrm{d}S \to 0 \quad \text{as} \quad R(\Omega) \to \infty. \tag{10}$$

3) $\mathrm{div}\,\mathsf{k} = 0$. [This is always true for the relevant field k, Eq. (8).]

Macro-homogeneous fields k and ∇p (continued)

It is easy to show that:

1) $\Rightarrow \overline{\mathrm{k}}^{\,\Omega} := \frac{1}{V(\Omega)} \int_\Omega \mathrm{k} \, \mathrm{d}\, V \to \mathrm{k}_0 \quad$ as $\quad R(\Omega) \to \infty.$

2) $\Rightarrow \overline{\nabla p}^{\,\Omega} \to \mathrm{g}_0 \quad$ as $\quad R(\Omega) \to \infty.$

Using that, one can prove that 1)-2)-3) imply that

$$\Delta_\Omega := \overline{\mathrm{k}.\nabla p}^{\,\Omega} - \overline{\mathrm{k}}^{\,\Omega} . \overline{\nabla p}^{\,\Omega} \to 0 \quad \text{as} \quad R(\Omega) \to \infty. \tag{11}$$

Thus, if k and ∇p are macro-homogeneous and $\operatorname{div} \mathrm{k} = 0$, we have in practice

$$\boxed{\overline{\mathrm{k}.\, \nabla p} = \overline{\mathrm{k}}\, .\, \overline{\nabla p}.} \tag{12}$$

Averaged PDE for locally macro-homogeneous fields k and ∇p

Using the latter eq (12), the PDE (7) averages to

$$\boxed{K^\nu P_{,\nu} = F,} \qquad \text{or} \quad \mathrm{K}.\nabla P = F \qquad \mathrm{K} := \overline{\mathrm{k}}, \quad P := \overline{p}, \quad F := \overline{f} \tag{13}$$

i.e., the same as (7), but with spacetime-averaged fields.

Those averages to be taken at a scale where k and ∇p are (approximately) macro-homogeneous. (Now $R(\Omega)$ can't be arbitrarily large.)

In view of the huge ratio here between the galactic scale and the micro-scale (typical wavelength and pseudoperiod), there is enough room.

Solving the PDE for P (or p)

The PDE (13) for P rewrites as the advection equation

$$\partial_T P + U^j \partial_j P = S, \tag{14}$$

where

$$S := cF/K^0, \qquad U^j := cK^j/K^0. \tag{15}$$

Therefore, on the characteristic curves

$$\boxed{\frac{d\,\mathrm{x}}{d\,T} = \mathrm{U}(T, \mathrm{x}), \qquad \mathrm{x}(T_0) = \mathrm{x}_0,} \tag{16}$$

we have

$$\frac{d\,P}{d\,T} = \frac{\partial P}{\partial T} + \frac{\partial P}{\partial x^j}\frac{d\,x^j}{d\,T} = S(T, \mathrm{x}), \tag{17}$$

so

$$\boxed{P(T, \mathrm{x}(T)) = P(T_0, \mathrm{x}_0) + \int_{T_0}^{T} S(t, \mathrm{x}(t))\,d\,t.} \tag{18}$$

Calculating the micro-field k^ν

– The 4-vector "micro-field" k^ν depends only on the (micro) EM field $(\mathbf{E}, \mathbf{B})$:

$$k^0 = \frac{-c}{(\mathbf{E}.\mathbf{B})^2} \mathbf{B}.\nabla(\mathbf{E}.\mathbf{B}), \qquad (19)$$

$$(k^i) = \frac{1}{(\mathbf{E}.\mathbf{B})^2} \left(\frac{\partial\,(\mathbf{E}.\mathbf{B})}{\partial T} \mathbf{B} - \mathbf{E} \wedge (\nabla(\mathbf{E}.\mathbf{B})) \right). \qquad (20)$$

To compute k^ν, we use the "Maxwell model of the interstellar radiation field", based on axial symmetry (of the galaxy and the ISRF) as a relevant approximation
(MA: Open Phys. **18**, 255 (2020); Open Phys. **19**, 77 (2021); Adv. Astron. **2021**, 5524600 (2021); Open Phys. **21**, 20220253 (2023)).

The averaged field K^ν

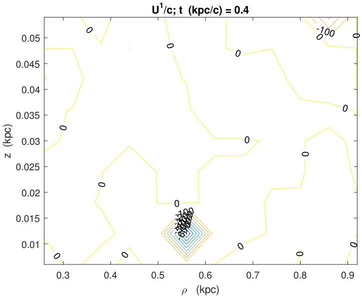
The micro-field k^ν is computed with the said model on a 3D spacetime "fine" grid
(axisymmetry $\Leftrightarrow$ independence of $\phi \Rightarrow$ variables $t,\ \rho,\ z$):

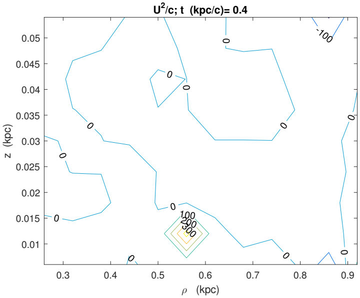
$$x_k^\mu = x_0^\mu + (k-1)\delta_\mu, \quad \mu = 0, 1, 2;\ k = 1, ..., N_\mu.$$

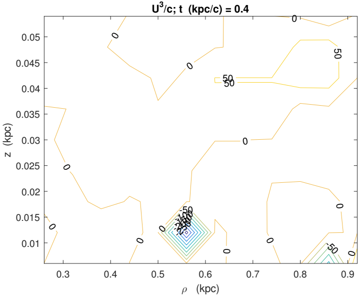
Then we take the local spacetime average K^ν of the field k^ν on a "rough" grid with steps $\delta_{\mu\, g} = g\delta_\mu$ (g integer, say $g = 6$).

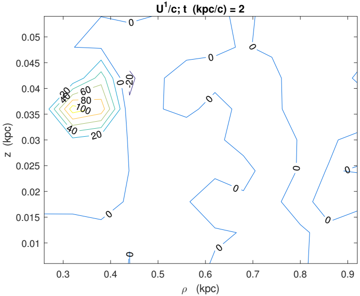
The average is done by considering, for each point of the rough grid, its $(2g+1)^3$ nearest neighbours of the fine grid, thus a discrete averaging.

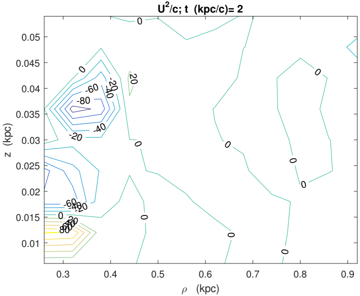
Then we calculate $U^j = cK^j/K^0$ whose integral lines are the characteristics.

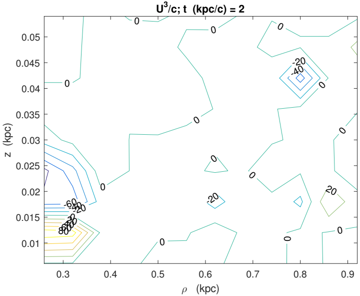

U1/c; t (kpc/c) = 0.4
z (kpc)
ρ (kpc)
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.3
0.4
0.5
0.6
0.7
0.8
0.9
-100


U2/c; t (kpc/c)= 0.4
z (kpc)
ρ (kpc)
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.3
0.4
0.5
0.6
0.7
0.8
0.9
-100
100
200
300


U^3/c; t (kpc/c) = 0.4
z (kpc)
ρ (kpc)


U^1/c; t (kpc/c) = 2
z (kpc)
ρ (kpc)


U²/c; t (kpc/c)= 2
z (kpc)
ρ (kpc)


U³/c; t (kpc/c) = 2
z (kpc)
ρ (kpc)

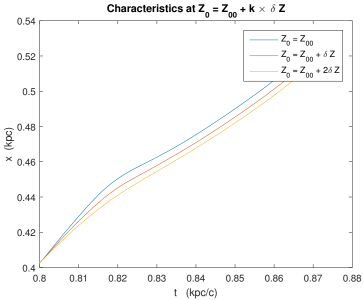
Characteristic curves

Currently we compute some characteristic curves (16):
We select two sets of initial conditions: $T = T_0$ and positions in galactic frame $\mathcal{E}_V$ (relative velocity V w.r.t. ether frame $\mathcal{E}$):

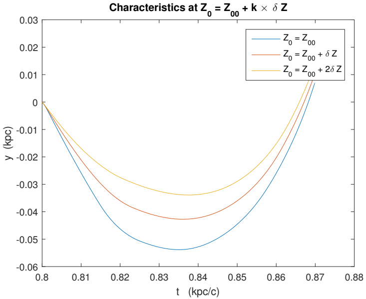
1) either ϕ_0 variable and $z'_0 = Z_0$ given:
$x'_0 = \rho_0 \cos(\phi_0), \; y'_0 = \rho_0 \sin(\phi_0), \quad \phi_0 = (i_\phi - 1) \times \frac{2\pi}{N_\phi}, \quad i_\phi = 1, ..., N_\phi,$

2) or z'_0 variable and $\phi_0 = 0$ given:
$x'_0 = \rho_0, \; y'_0 = 0, \; z'_0 = Z_0 + (i_Z - 1)\delta Z, \quad i_Z = 1, ..., N_Z.$

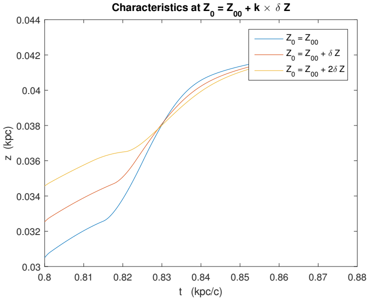
In both cases, positions x_0, y_0, z_0 in ether frame $\mathcal{E}$ by Lorentz transformation, imposing time of $\mathcal{E}$ is $T = T_0$. Note that $k^\nu := G^{\mu\nu}_{\ ,\mu}$ is a 4-vector.
We then numerically integrate the ODE (16) for the characteristic curves.

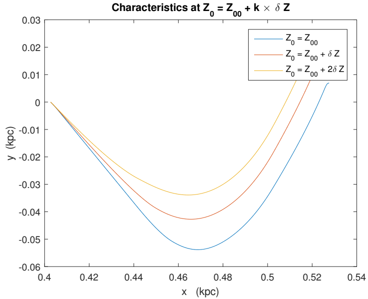


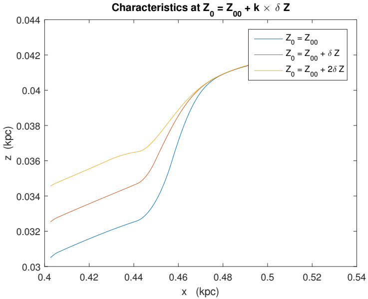
Characteristics at Z0 = Z00 + k × δ Z
Z0 = Z00
Z0 = Z00 + δ Z
Z0 = Z00 + 2δ Z
x (kpc)
t (kpc/c)

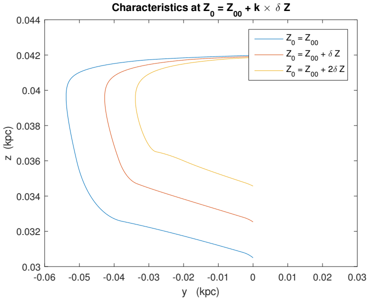


Characteristics at Z0 = Z00 + k × δ Z
Z0 = Z00
Z0 = Z00 + δ Z
Z0 = Z00 + 2δ Z
y (kpc)
t (kpc/c)


Characteristics at $Z_0 = Z_{00} + k \times \delta Z$
$Z_0 = Z_{00}$
$Z_0 = Z_{00} + \delta Z$
$Z_0 = Z_{00} + 2\delta Z$
z (kpc)
t (kpc/c)
0.044
0.042
0.04
0.038
0.036
0.034
0.032
0.03
0.8
0.81
0.82
0.83
0.84
0.85
0.86
0.87
0.88


Characteristics at $Z_0 = Z_{00} + k \times \delta Z$
$Z_0 = Z_{00}$
$Z_0 = Z_{00} + \delta Z$
$Z_0 = Z_{00} + 2\delta Z$
y (kpc)
x (kpc)
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
0.4
0.42
0.44
0.46
0.48
0.5
0.52
0.54


Characteristics at Z0 = Z00 + k × δ Z
Z0 = Z00
Z0 = Z00 + δ Z
Z0 = Z00 + 2δ Z
z (kpc)
x (kpc)
0.03
0.032
0.034
0.036
0.038
0.04
0.042
0.044
0.4
0.42
0.44
0.46
0.48
0.5
0.52
0.54


Characteristics at Z0 = Z00 + k × δ Z
Z0 = Z00
Z0 = Z00 + δ Z
Z0 = Z00 + 2δ Z
z (kpc)
y (kpc)
0.03
0.032
0.034
0.036
0.038
0.04
0.042
0.044
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03

Conclusion

In the alternative gravity theory "SET", electromagnetism in the presence of gravitation demands to introduce an additional energy tensor $\boldsymbol{T}_{\text{inter}}$, depending on a scalar field p.

This exotic energy might contribute to dark matter. The PDE (7) was derived: governs the field p in given EM + gravity fields.

Developed a model that provides the EM field in a galaxy.

Quick variation of EM field prevents integration of (7) in a galaxy. Homogenization theory not found to provide tractable results.

Using the theory of macro-homogeneous fields, proved that the PDE stays unchanged but with spacetime-averaged fields.

Currently able to compute characteristic lines at sub-kpc scale.