Relativistic single-electron wavepackets in electromagnetic vacuum: Quantum coherence and the Unruh effect

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[SYL & BL Hu, JHEP 04(2024)065]
[SYL, IARD2022, JPCS 2482(2023)012018]

5 Jun 2024, IARD @ Aalto U

Unruh effect

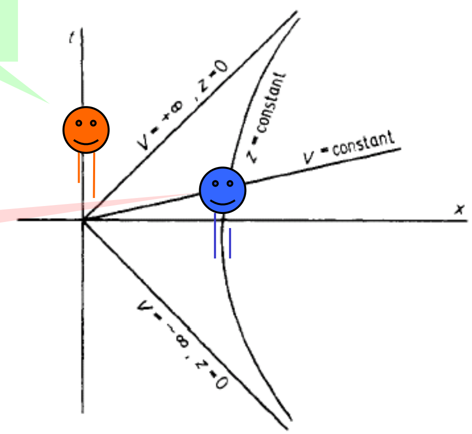
Vacuum is not vacuous [Unruh, PRD14(1976)870]

A “detector” linearly, uniformly accelerated in Minkowski vacuum will experience a thermal bath at the **Unruh temperature**:

T_U = 1K for
a = 2.4 x 10^{20} m/s^2

$$T_U = \frac{\hbar a}{2\pi k_B c}$$

T = 0



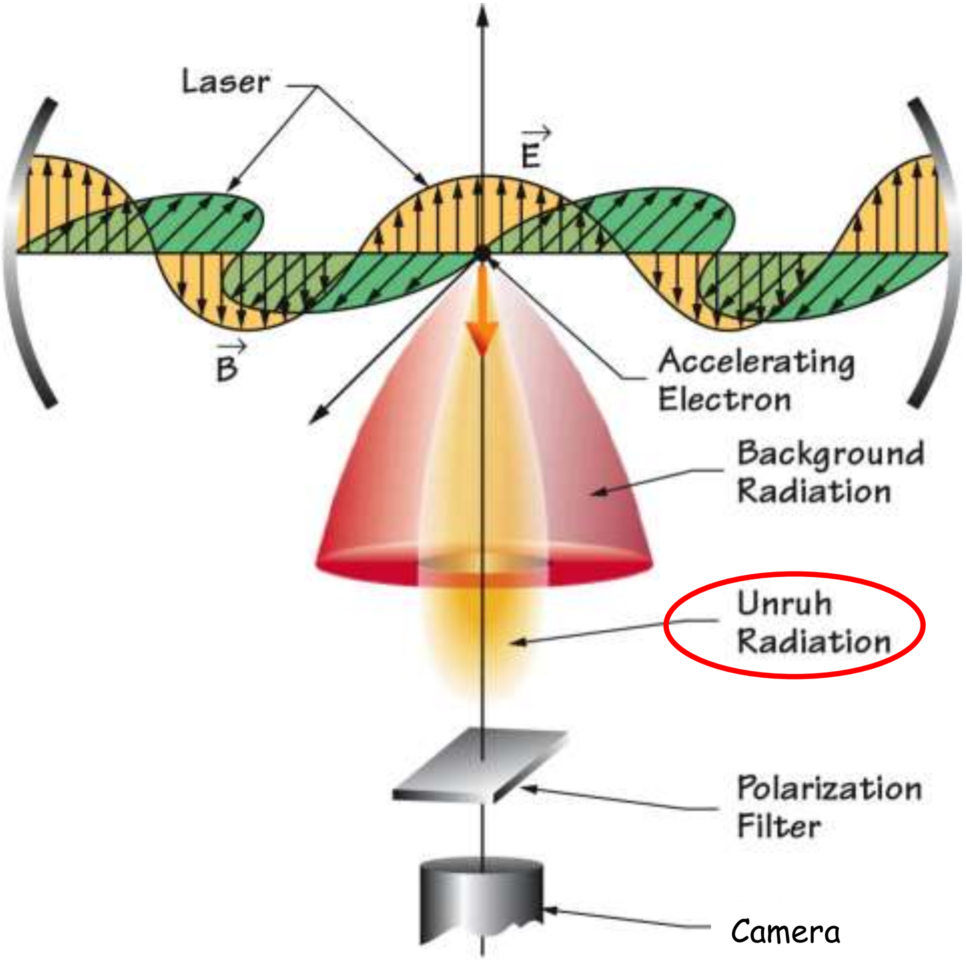
** If [smiley] is initially in its ground state, <u>in weak-coupling limit</u>, the transition probability to its 1st excited state is*

$$\rho^R_{1,1}\big|_{\gamma\eta\to 0} \xrightarrow{\eta\gg a^{-1}} \frac{\lambda_0^2}{4\pi m_0}\left[\frac{\eta}{e^{2\pi\Omega_r/a}-1}\right]$$

- linear in $\eta \equiv \tau - \tau_0$

- uniform acceleration : $a_m a^m = a^2 = \text{constant}$ (a: proper acceleration)
- Minkowski vacuum: <u>No particle</u> (field quanta) state of the field for Minkowski observer

A Conceptual Design of an Experiment for Detecting the Unruh Effect



Schematic Diagram for Detecting Unruh Radiation

[Chen, Tajima, PRL83('99)256]

The 10^{13} W (10 TW) laser in NTU, focused on a spot of 10^{-6} cm^2 can produce

$a \sim 3 \times 10^{24}$ m/s^2

$T_U \sim 7 \times 10^4$ K

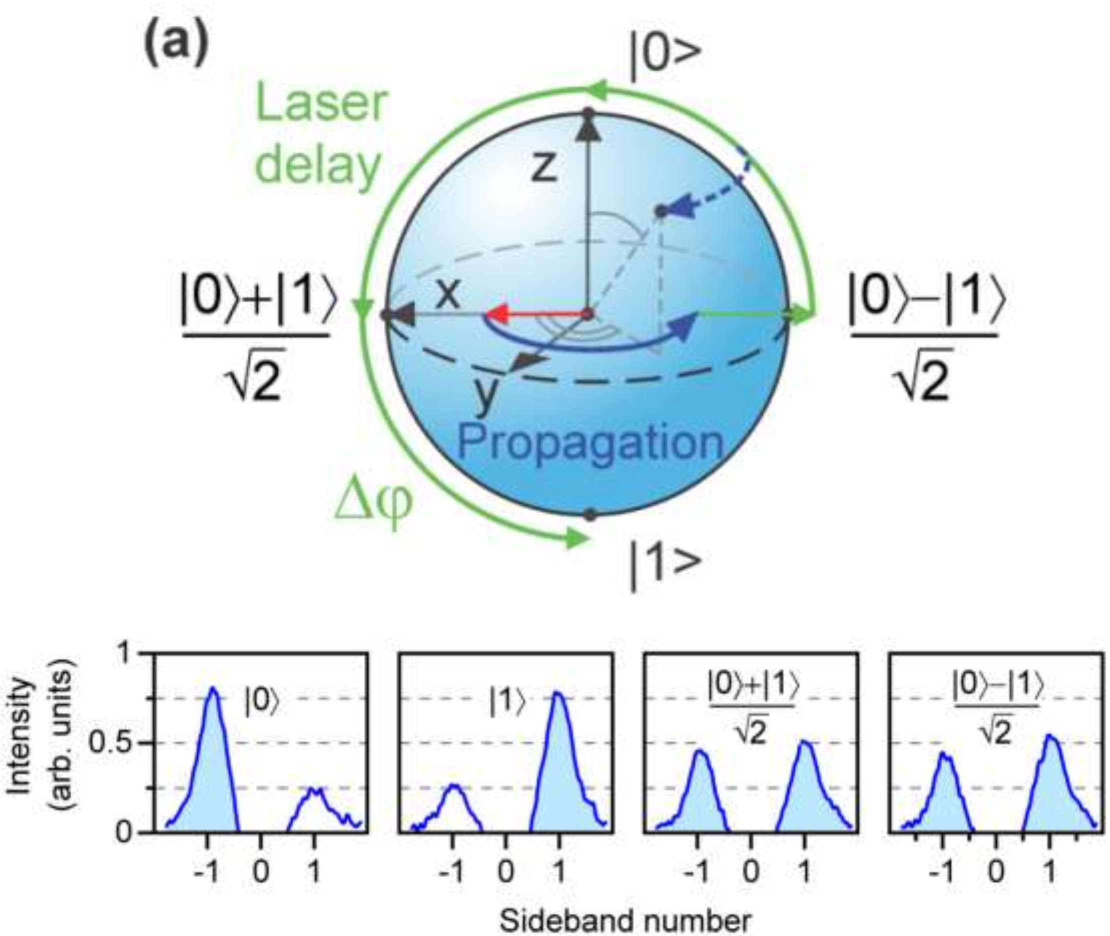
on an electron.

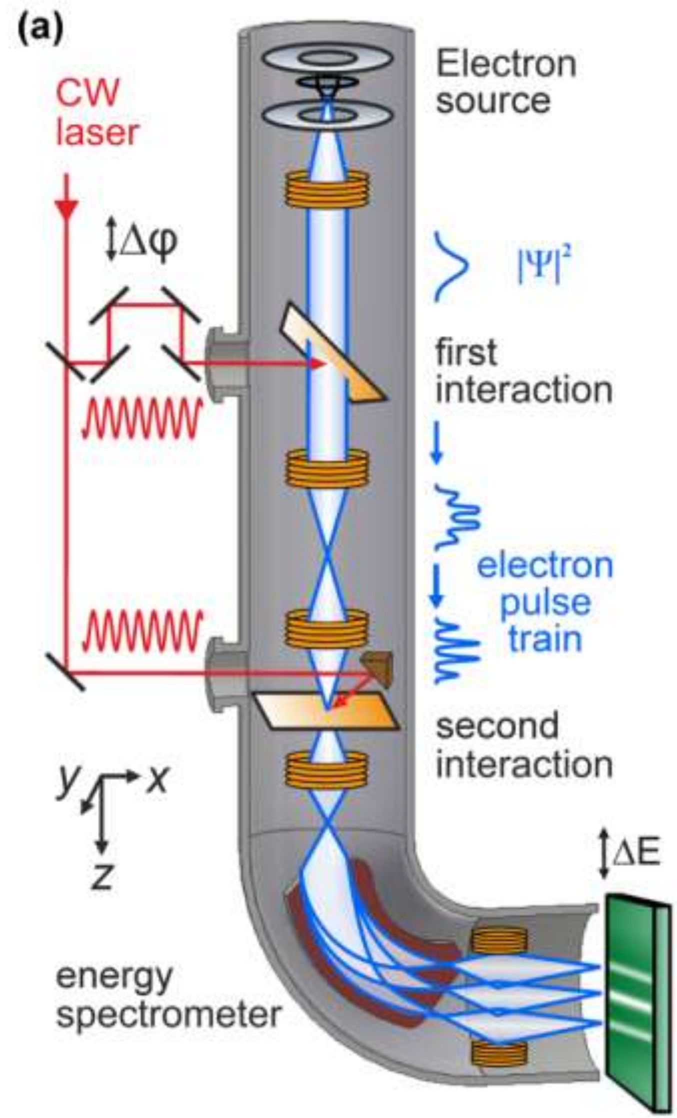
Note: The most powerful laser up to 2024 (e.g. ELI-NP) can reach the intensity 10^{24} W/cm^2, which is well below 10^{29} W/cm^2 (Schwinger limit.)

[Courtesy of Pisin Chen]

Free electron qubit

- [Tsarev, Ryabov, Baum, PRR3(2021)043033]





“Free electron quantum optics”

Outline

I. Introduction

II. Single-electron wavepackets in Quantum Electromagnetic Fields

III. Correlators, regulators, and quantum coherence

IV. Unruh effect

V. Summary

II. Single-Electron Wavepackets in Quantum Electromagnetic Fields

Charged Particle in EM Fields

- $z^\mu(\tau)$ and $A^\mu(x)$ are dynamical variables

$$S = -mc\int d\tau\sqrt{-\frac{dz_\mu}{d\tau}\frac{dz^\mu}{d\tau}} + \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + j_\mu(x)A^\mu(x)\right]$$

$$j_\mu(x) \equiv e\int d\tau\, v_\mu(\tau)\,\delta^4(x^\mu - z^\mu(\tau))$$

- Cf: Unruh-DeWitt "detector" with internal HO

$S = S_Q + S_\Phi + S_I,$ where

$S_Q = \int d\tau \frac{m_0}{2}\left[(\partial_\tau Q)^2 - \Omega_0^2 Q^2\right]$	*Internal: harmonic oscillator*
$S_\Phi = -\int d^4x \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi$	*Massless scalar field*
$S_I = \lambda_0\int d\tau\int d^4x Q(\tau)\Phi(x)\underline{\delta^4\left(x^\mu - z^\mu(\tau)\right)}$	*Point-like object* [DeWitt 1979]

prescribed worldline

Charged Particle in EM Fields

- $z^i(t)$ and $A^\mu(t, \mathbf{x})$ are dynamical variables

$S = S_z + S_I + S_F$ where

$$S_z = -mc \int d\tau \sqrt{-\frac{dz_\mu}{d\tau}\frac{dz^\mu}{d\tau}} = -mc^2 \int dt \sqrt{1 - \frac{1}{c^2}\frac{dz_i}{dt}\frac{dz^i}{dt}}$$

$$S_I = q \int d^4x \int d\tau \frac{dz^\mu}{d\tau} \delta^4\left[x - z(\tau)\right] A_\mu(x)$$

$$= \int dt \left[qcA_0(t, \mathbf{z}(t)) + q\frac{dz^i}{dt} A_i(t, \mathbf{z}(t)) \right]$$

$$S_F = \int \frac{dt d^3x}{\mu_0} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{2} \left(\partial_\mu A^\mu\right)^2 \right]$$

have chosen the Minkowski-time gauge

Gupta-Bleuler quantization in Lorentz/Feynman gauge

Highly nonlinear!

Linearization

- Introduce classical fields and trajectory as background

$$A^\mu(t,\mathbf{x}) = \bar{A}^\mu(t,\mathbf{x}) + \underline{\tilde{A}^\mu(t,\mathbf{x})}$$

classical fields *fluctuations*

(Solution to classical field equations)

$$\delta S/\delta A^\mu_{\mathbf{x}}(t) = 0$$

$$z^i(t) = \bar{z}^i(t) + \underline{\tilde{z}^i(t)}$$

classical trajectory *deviations*

(Solution to classical eq. of motion)

$$\delta S/\delta z^i(t) = 0$$

Only the field fluctuations and trajectory deviations will be quantized.

- Expand the action to the quadratic order

$$S \approx S[\bar{z},\bar{A}] + \frac{1}{2}\int dt\,dt' \left\{ \sum_{i,j} \tilde{z}^i(t) \left.\frac{\delta^2 S}{\delta z^i(t)\delta z^j(t')}\right|_{\bar{z},\bar{A}} \tilde{z}^j(t') + \right.$$

$$\left. 2\sum_{i,\mu,\mathbf{x}} \tilde{z}^i(t) \left.\frac{\delta^2 S}{\delta z^i(t)\delta A^\mu_{\mathbf{x}}(t')}\right|_{\bar{z},\bar{A}} \tilde{A}^\mu_{\mathbf{x}}(t') + \sum_{\mu,\mathbf{x},\nu,\mathbf{y}} \tilde{A}^\nu_{\mathbf{y}}(t) \left.\frac{\delta^2 S}{\delta A^\nu_{\mathbf{y}}(t)\delta A^\mu_{\mathbf{x}}(t')}\right|_{\bar{z},\bar{A}} \tilde{A}^\mu_{\mathbf{x}}(t') \right\}$$

Linearization

- Canonical conjugate momenta

$$\tilde{p}_i = \frac{\delta S}{\delta \partial_t \tilde{z}^i} = m\bar{\gamma}\bar{M}_{ij}\dot{\tilde{z}}^j + \frac{q}{c}\tilde{z}^j \partial_j \bar{A}_i^{\bar{\mathbf{z}}},$$

$$\tilde{\pi}_{\mathbf{x}}^i = \frac{\delta S}{\delta \partial_t \tilde{A}_i^{\mathbf{x}}} = \frac{1}{c}\tilde{F}_{\mathbf{x}}^{i0} - \frac{q}{c}\tilde{z}^i \delta^3(\mathbf{x} - \bar{\mathbf{z}}),$$

$$\tilde{\pi}_{\mathbf{x}}^0 = \frac{\delta S}{\delta \partial_t \tilde{A}_0^{\mathbf{x}}} = \frac{\alpha}{c}\partial_\mu \tilde{A}_{\mathbf{x}}^\mu,$$

- Hamiltonian of the quadratic action

$$\begin{aligned}
\tilde{H}_2 &= \tilde{p}_i \dot{\tilde{z}}^i + c\sum_{\mathbf{x}} \left(\tilde{\pi}_{\mathbf{x}}^i \partial_0 \tilde{A}_i^{\mathbf{x}} + \tilde{\pi}_{\mathbf{x}}^0 \partial_0 \tilde{A}_0^{\mathbf{x}} \right) - L_2 \\
&= \frac{\bar{M}^{ij}}{2m\bar{\gamma}} \left(\tilde{p}_i - \frac{q}{c}\tilde{z}^k \partial_k \bar{A}_i^{\bar{\mathbf{z}}} \right) \left(\tilde{p}_j - \frac{q}{c}\tilde{z}^l \partial_l \bar{A}_j^{\bar{\mathbf{z}}} \right) - \frac{q\bar{v}^j}{c}\tilde{z}^i \tilde{F}_{ij}^{\bar{\mathbf{z}}} - \frac{q\bar{v}^\mu}{2c}\tilde{z}^i \tilde{z}^j \partial_i \partial_j \bar{A}_\mu^{\bar{\mathbf{z}}} \\
&\quad + \sum_{\mathbf{x}} \left\{ \frac{1}{2} \left[c\tilde{\pi}_i^{\mathbf{x}} + q\tilde{z}_i \delta^3(\mathbf{x} - \bar{\mathbf{z}}) \right] \left[c\tilde{\pi}_{\mathbf{x}}^i + q\tilde{z}^i \delta^3(\mathbf{x} - \bar{\mathbf{z}}) \right] \right. \\
&\qquad \left. - \frac{c^2}{2\alpha} \left(\tilde{\pi}_{\mathbf{x}}^0 \right)^2 + c\tilde{\pi}_{\mathbf{x}}^i \partial_i \tilde{A}_0^{\mathbf{x}} + c\tilde{\pi}_{\mathbf{x}}^0 \partial_i \tilde{A}_{\mathbf{x}}^i + \frac{1}{4}\tilde{F}_{ij}^{\mathbf{x}} \tilde{F}_{\mathbf{x}}^{ij} \right\}
\end{aligned}$$

Quantization

- Promote the perturbative variables $\tilde{z}^i$ and $\tilde{A}^i_{\mathbf{x}}$ to the operators $\hat{z}^i$ and $\hat{A}^i_{\mathbf{x}}$ and introduce the quantization conditions in the Lorentz gauge,

$$[\hat{z}^i, \hat{p}_j] = i\hbar\delta^i_j, \qquad [\hat{A}^\mu_{\mathbf{x}}, \hat{\pi}^\nu_{\mathbf{y}}] = i\hbar\eta^{\mu\nu}\delta^3(\mathbf{x}-\mathbf{y}).$$

QM of a single electron x QFT of EM fields

In this talk - system x environment

- Hamiltonian of the quadratic action

$$\begin{aligned}
\tilde{H}_2 &= \tilde{p}_i\dot{\tilde{z}}^i + c\sum_{\mathbf{x}}\left(\tilde{\pi}^i_{\mathbf{x}}\partial_0\tilde{A}^{\mathbf{x}}_i + \tilde{\pi}^0_{\mathbf{x}}\partial_0\tilde{A}^{\mathbf{x}}_0\right) - L_2 \\
&= \frac{\bar{M}^{ij}}{2m\bar{\gamma}}\left(\tilde{p}_i - \frac{q}{c}\tilde{z}^k\partial_k\bar{A}^{\bar{\mathbf{z}}}_i\right)\left(\tilde{p}_j - \frac{q}{c}\tilde{z}^l\partial_l\bar{A}^{\bar{\mathbf{z}}}_j\right) - \frac{q\bar{v}^j}{c}\tilde{z}^i\tilde{F}^{\bar{\mathbf{z}}}_{ij} - \frac{q\bar{v}^\mu}{2c}\tilde{z}^i\tilde{z}^j\partial_i\partial_j\bar{A}^{\bar{\mathbf{z}}}_\mu \\
&\quad + \sum_{\mathbf{x}}\left\{\frac{1}{2}\left[c\tilde{\pi}^{\mathbf{x}}_i + q\tilde{z}_i\delta^3(\mathbf{x}-\bar{\mathbf{z}})\right]\left[c\tilde{\pi}^i_{\mathbf{x}} + q\tilde{z}^i\delta^3(\mathbf{x}-\bar{\mathbf{z}})\right]\right. \\
&\qquad \left. - \frac{c^2}{2\alpha}\left(\tilde{\pi}^0_{\mathbf{x}}\right)^2 + c\tilde{\pi}^i_{\mathbf{x}}\partial_i\tilde{A}^{\mathbf{x}}_0 + c\tilde{\pi}^0_{\mathbf{x}}\partial_i\tilde{A}^i_{\mathbf{x}} + \frac{1}{4}\tilde{F}^{\mathbf{x}}_{ij}\tilde{F}^{ij}_{\mathbf{x}}\right\}
\end{aligned}$$

Mode Functions

- In our linear quantum theory,

  Heisenberg equations for the operators $\hat{z}^i$ and $\hat{A}^i_{\mathbf{x}}$

  ~ Hamilton equations for the variables $\tilde{z}^i$ and $\tilde{A}^i_{\mathbf{x}}$

  ~ Equations for the mode functions $\mathcal{Z}^j_{\Omega}$ and $\mathcal{A}^{\mu}_{\Omega}(t,\mathbf{x})$

- Operator expansion and mode functions

$$\hat{z}^i(t) = \sum_{j=1}^{3}\left[\underline{\mathcal{Z}^i_{z^j}}(t)\hat{z}^j + \underline{\mathcal{Z}^i_{p_j}}(t)\hat{p}_j\right] + \sum_{\mathbf{k}}\sum_{\lambda=0}^{3}\left[\underline{\mathcal{Z}^i_{(\lambda)\mathbf{k}}}(t)\hat{b}^{(\lambda)}_{\mathbf{k}} + \underline{\mathcal{Z}^{i*}_{(\lambda)\mathbf{k}}}(t)\hat{b}^{(\lambda)\dagger}_{\mathbf{k}}\right]$$

$$\hat{A}^{\mu}_{\mathbf{x}}(t) = \sum_{j=1}^{3}\left[\underline{\mathcal{A}^{\mu}_{z^j}(t,\mathbf{x})}\hat{z}^j + \underline{\mathcal{A}^{\mu}_{p_j}(t,\mathbf{x})}\hat{p}_j\right] + \sum_{\mathbf{k}}\sum_{\lambda=0}^{3}\left[\underline{\mathcal{A}^{\mu}_{(\lambda)\mathbf{k}}(t,\mathbf{x})}\hat{b}^{(\lambda)}_{\mathbf{k}} + \underline{\mathcal{A}^{\mu*}_{(\lambda)\mathbf{k}}(t,\mathbf{x})}\hat{b}^{(\lambda)\dagger}_{\mathbf{k}}\right]$$

Self Fields and Back Reaction

"Particle" eq. $\partial_t\left(m\bar{\gamma}\bar{M}_{ij}\dot{\mathcal{Z}}^j_\Omega\right) = q\left[\mathcal{F}^{\bar{\mathbf{z}}}_{\Omega i\mu}\bar{v}^\mu + \mathcal{Z}^j_\Omega\left(\partial_j\bar{F}^{\bar{\mathbf{z}}}_{i\mu}\right)\bar{v}^\mu + \bar{F}^{\bar{\mathbf{z}}}_{ij}\dot{\mathcal{Z}}^j_\Omega\right]$

"Field" eq. $\partial_\mu\mathcal{F}^{\mu\nu}_\Omega(t,\mathbf{x}) + \bar{\alpha}\partial^\nu\partial_\mu\mathcal{A}^\mu_\Omega(t,\mathbf{x}) = -\mu_0 q\mathcal{V}^\nu_\Omega\delta^3(\mathbf{x}-\bar{\mathbf{z}})$

$$\mathcal{F}^\Omega_{\mu\nu} = \partial_\mu\mathcal{A}^\Omega_\nu - \partial_\nu\mathcal{A}^\Omega_\nu$$

$$\mathcal{V}^0_\Omega(t) \equiv -c\mathcal{Z}^j_\Omega\partial_j$$

$$\mathcal{V}^i_\Omega(t) \equiv \partial_t\mathcal{Z}^i_\Omega - \mathcal{Z}^j_\Omega\bar{v}^i\partial_j$$

$$\mathcal{A}^\mu_\Omega(t,\mathbf{x}) = \mathcal{A}^\mu_{[0]\Omega}(t,\mathbf{x}) + \mathcal{A}^\mu_{[1]\Omega}(t,\mathbf{x})$$

homogeneous solution ("external" fields)

$$\mathcal{A}^\mu_{[1]\Omega}(t,\mathbf{x}) = q\int dt'd^3x'\, G_{ret}(t,\mathbf{x};t',\mathbf{x}')\mathcal{V}^\mu_\Omega(t')\delta^3(\mathbf{x}'-\bar{\mathbf{z}}(t'))$$

In the Lorentz-Feynman gauge $\alpha = 1$

Self Fields and Back Reaction

- Quantum analogy of Lorentz-Abraham-Dirac (LAD) equation

  Particle-deviation equation with self force and back reaction ($t - t_0 \gg (c\Lambda)^{-1}$)

$$\bar{m}\partial_t \left\{ \bar{\gamma}\bar{M}_{ij}\partial_t \mathcal{Z}^j_\Omega(t) \right\} = \sum_{n=1}^{3} \frac{\partial \bar{\Gamma}_i}{\partial\left(\partial_t^n \bar{z}^j\right)} \partial_t^n \mathcal{Z}^j_\Omega + q\left\{ \bar{v}^\mu \mathcal{F}^{[0]\Omega}_{i\mu}(t, \bar{\mathbf{z}}(t)) + \bar{v}^\mu \mathcal{Z}^j_\Omega \partial_j \bar{F}^{[0]}_{i\mu}(t, \bar{\mathbf{z}}) + \bar{F}^{[0]}_{ij}(t, \bar{\mathbf{z}})\partial_t \mathcal{Z}^j_\Omega \right\} + O(\Lambda^{-1})$$

field fluctuations

Self Fields and Back Reaction

- Quantum analogy of Lorentz-Abraham-Dirac (LAD) equation

Particle-deviation equation with self force and back reaction ($t - t_0 \gg (c\Lambda)^{-1}$)

$$\bar{m}\partial_t\left\{\bar{\gamma}\bar{M}_{ij}\partial_t\mathcal{Z}^j_\Omega(t)\right\} = \sum_{n=1}^{3}\frac{\partial\bar{\Gamma}_i}{\partial\left(\partial^n_t\bar{z}^j\right)}\partial^n_t\mathcal{Z}^j_\Omega + q\left\{\bar{v}^\mu\mathcal{F}^{[0]\Omega}_{i\mu}(t,\bar{\mathbf{z}}(t)) + \bar{v}^\mu\mathcal{Z}^j_\Omega\partial_j\bar{F}^{[0]}_{i\mu}(t,\bar{\mathbf{z}}) + \bar{F}^{[0]}_{ij}(t,\bar{\mathbf{z}})\partial_t\mathcal{Z}^j_\Omega\right\} + O(\Lambda^{-1})$$

with the renormalized mass

$$\bar{m} \equiv m + \Delta_m, \qquad \Delta_m \equiv \frac{\mu_0 q^2 2^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}{4\pi\sqrt{\pi}}\Lambda \qquad \Lambda = 1/\lambda_C = mc/h$$

Self Fields and Back Reaction

- Quantum analogy of Lorentz-Abraham-Dirac (LAD) equation

Particle-deviation equation with self force and back reaction ($t - t_0 \gg (c\Lambda)^{-1}$)

$$\bar{m}\partial_t\left\{\bar{\gamma}\bar{M}_{ij}\partial_t\mathcal{Z}^j_\Omega(t)\right\} = \sum_{n=1}^{3}\frac{\partial\bar{\Gamma}_i}{\partial\left(\partial_t^n\bar{z}^j\right)}\partial_t^n\mathcal{Z}^j_\Omega + q\left\{\bar{v}^\mu\mathcal{F}^{[0]\Omega}_{i\mu}(t,\bar{\mathbf{z}}(t)) + \bar{v}^\mu\mathcal{Z}^j_\Omega\partial_j\bar{F}^{[0]}_{i\mu}(t,\bar{\mathbf{z}}) + \bar{F}^{[0]}_{ij}(t,\bar{\mathbf{z}})\partial_t\mathcal{Z}^j_\Omega\right\} + O(\Lambda^{-1})$$

with the renormalized mass

$$\bar{m} \equiv m + \Delta_m, \qquad \Delta_m \equiv \frac{\mu_0 q^2 2^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}{4\pi\sqrt{\pi}}\Lambda$$

$\Lambda = 1/\lambda_C = mc/h$

and the counterpart of the LAD force

$$\sum_{n=1}^{3}\frac{\partial\bar{\Gamma}_i}{\partial\left(\partial_t^n\bar{z}^j\right)}\partial_t^n\mathcal{Z}^j_\Omega = \mu_0\frac{q^2\bar{\gamma}^4}{4\pi c^3}\times$$
$$\left\{\frac{2c^2}{3\bar{\gamma}^2}\bar{M}_{ij}\partial_t^3\mathcal{Z}^j_\Omega + 2\left[\bar{v}^k\dot{\bar{v}}_k\eta_{ij} + \left(\dot{\bar{v}}_i + 2\frac{\bar{\gamma}^2}{c^2}\bar{v}^k\dot{\bar{v}}_k\bar{v}_i\right)\bar{v}_j\right]\partial_t^2\mathcal{Z}^j_\Omega\right.$$
$$+2\left[\frac{1}{3}\bar{v}^k\ddot{\bar{v}}_k\eta_{ij} + \frac{\bar{\gamma}^2}{c^2}\left(\bar{v}^k\dot{\bar{v}}_k\right)^2\eta_{ij} + \frac{1}{3}\bar{v}_i\ddot{\bar{v}}_j + \frac{2}{3}\ddot{\bar{v}}_i\bar{v}_j + \frac{4\bar{\gamma}^2}{3c^2}\bar{v}^k\ddot{\bar{v}}_k\bar{v}_i\bar{v}_j\right.$$
$$\left.\left. + \dot{\bar{v}}_i\dot{\bar{v}}_j + \frac{\bar{\gamma}^2}{c^2}\bar{v}^k\dot{\bar{v}}_k\left(4\dot{\bar{v}}_i\bar{v}_j + 2\bar{v}_i\dot{\bar{v}}_j\right) + 6\left(\frac{\bar{\gamma}^2}{c^2}\bar{v}^k\dot{\bar{v}}_k\right)^2\bar{v}_i\bar{v}_j\right]\partial_t\mathcal{Z}^j_\Omega\right\}$$

$s \equiv \frac{q^2\mu_0}{6\pi c\bar{m}} \approx$ 6.3×10^{-24} s for electrons.

$3cs/2 = r_0$ - classical electron radius ~ 2.8 x 10^{-15} m

III. Correlators, regulators, and quantum coherence

Gaussian Approximation

- Assume the initial state of the combined system is a Gaussian state = (Gaussian wave packet of the particle) x (Minkowski vacuum of the fields)

Gaussianity of quantum state can always be preserved in our linear theory, and the (reduced) state is fully determined by the **symmetric two-point correlators**.
e.g. the reduced state of a **particle in 1D** at τ reads

$$\rho^R(Q, Q'; \tau) = \int \mathcal{D}\Phi_k \psi_0[Q, \Phi_k; \tau]\psi_0^*[Q', \Phi_k; \tau]$$
$$= \exp[-\underline{G^{ij}(\tau)}Q_iQ_j - \underline{F(\tau)}],$$

normalization

where $i, j = 1,\ 2,\ Q_i = (Q, Q')$,

$$G^{11} + G^{22} + 2G^{12} = \frac{1}{2\langle Q^2\rangle},$$
$$G^{11} + G^{22} - 2G^{12} = \frac{2}{\hbar^2\langle Q^2\rangle}[\langle P^2\rangle\langle Q^2\rangle - \langle P, Q\rangle^2],$$
$$G^{11} - G^{22} = -\frac{i\langle P, Q\rangle}{\hbar\langle Q^2\rangle}$$

Quantum coherence

- Purity of the reduced state of our charged particle in 3D,

$$\mathbf{P} = \mathrm{Tr}\,(\rho^R \rho^R) = \frac{(\hbar/2)^3}{\mathcal{U}}$$

where $\mathcal{U}(\tau) \equiv \sqrt{|\det \mathbf{C}|}$ uncertainty function

$$\mathbf{C} = \begin{pmatrix} \langle \hat{p}_i(\tau), \hat{p}_j(\tau)\rangle & \langle \hat{z}^i(\tau), \hat{p}_j(\tau)\rangle \\ \langle \hat{p}_i(\tau), \hat{z}^j(\tau)\rangle & \langle \hat{z}^i(\tau), \hat{z}^j(\tau)\rangle \end{pmatrix}$$ covariance matrix (6x6)

In the following two cases,

$$\mathcal{U} = \prod_{i=1}^{3} \sqrt{u_i}, \qquad u_i \equiv \langle \hat{p}_i(t), \hat{p}_i(t)\rangle \langle \hat{z}_i(t), \hat{z}_i(t)\rangle - \langle \hat{p}_i(t), \hat{z}_i(t)\rangle^2$$

so we define $\mathbf{P}_i \equiv \dfrac{\hbar/2}{\sqrt{u_i}}$ in each direction.

Symmetric Two-Point Correlators

- Initial state: $\rho^{\mathbf{I}} = \rho^{\mathbf{I}}_{P} \otimes \rho^{\mathbf{I}}_{F}$ with $\rho^{\mathbf{I}}_{F} = |0_M\rangle\langle 0_M|$

  (Gaussian state of Particle) x (Minkowski vacuum of the Field)

- Symmetric particle correlators, e.g.,

$$\langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle \equiv \mathrm{Tr}\left[\rho^{\mathbf{I}}\left\{\hat{z}^j(t), \hat{z}^{j'}(t)\right\}\right] = \langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle_P + \langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle_F$$

"P(article)-part" $\langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle_P$

$$\equiv \mathrm{Tr}\left[\rho^{\mathbf{I}}_P \sum_{l,l'}\left\{\left(\mathcal{Z}^j_{z^l}(t)\hat{z}^l + \mathcal{Z}^j_{p_l}(t)\hat{p}_l\right), \left(\mathcal{Z}^{j'}_{z^{l'}}(t)\hat{z}^{l'} + \mathcal{Z}^{j'}_{p_{l'}}(t)\hat{p}_{l'}\right)\right\}\right]$$

"F(ield)-part" $\langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle_F$

$$\equiv \lim_{t'\to t}\sum_{\mathbf{k},\mathbf{k}'}\frac{1}{2}\left(\mathcal{Z}^j_{(\lambda)\mathbf{k}}(t)\mathcal{Z}^{j'*}_{(\lambda')\mathbf{k}'}(t') + \mathcal{Z}^{j'}_{(\lambda)\mathbf{k}}(t')\mathcal{Z}^{j*}_{(\lambda')\mathbf{k}'}(t)\right)\langle 0_M|\hat{b}^{(\lambda)}_{\mathbf{k}}\hat{b}^{(\lambda')\dagger}_{\mathbf{k}'}|0_M\rangle$$

$$\sum_{\mathbf{k}} \equiv \int \frac{d^3k}{(2\pi)^3}\sqrt{\frac{\hbar}{2\omega\varepsilon_0}}$$

$$\langle 0_M|\hat{b}^{(\lambda)}_{\mathbf{k}}\hat{b}^{(\lambda')\dagger}_{\mathbf{k}'}|0_M\rangle = (2\pi)^3\eta^{(\lambda)(\lambda')}\delta^3(\mathbf{k}-\mathbf{k}')$$

Divergences and Regularization

- The *F-part* correlators often diverge in the coincidence limit, *e.g.*,

$$\langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle_F \equiv \lim_{t'\to t} \sum_{\mathbf{k},\mathbf{k}'} \frac{1}{2} \left(\mathcal{Z}^j_{(\lambda)\mathbf{k}}(t) \mathcal{Z}^{j'*}_{(\lambda')\mathbf{k}'}(t') + \mathcal{Z}^{j'}_{(\lambda)\mathbf{k}}(t') \mathcal{Z}^{j*}_{(\lambda')\mathbf{k}'}(t) \right) \langle 0 | \hat{b}^{(\lambda)}_{\mathbf{k}} \hat{b}^{(\lambda')\dagger}_{\mathbf{k}'} | 0 \rangle$$

$$\propto \lim_{t',t'_0 \to t,t_0} \int_{\tau(t_0)}^{\tau(t)+\epsilon_1} d\tilde{\tau} K(\tau(t),\tilde{\tau}) \int_{\tau(t'_0)+\epsilon_0}^{\tau(t')} d\tilde{\tau}' K(\tau(t')-\tilde{\tau}')$$

$$\int_0^\infty \frac{\omega^2 e^{-\omega\epsilon}}{(2\pi)^3 2\omega} d\omega \int d\Omega\, \mathcal{E}^j_{(\lambda)\mathbf{k}} \mathcal{E}^{(\lambda)j'}_{\mathbf{k}} \left[e^{-i\omega t + i\omega t'}(\cdots) + \cdots \right]$$

Regulators:

- Suppression of short-wavelength contribution : $c\epsilon \sim \lambda_C / \gamma$,

with the electron Compton wavelength $\lambda_C = 2.4 \times 10^{-12}$ m, or

$\epsilon = t_C / \gamma$ t_C = 8.1 x 10^{-21} s : Compton time

cf. [Bethe, PR72 (1947) 339]

Gaussian Approximation

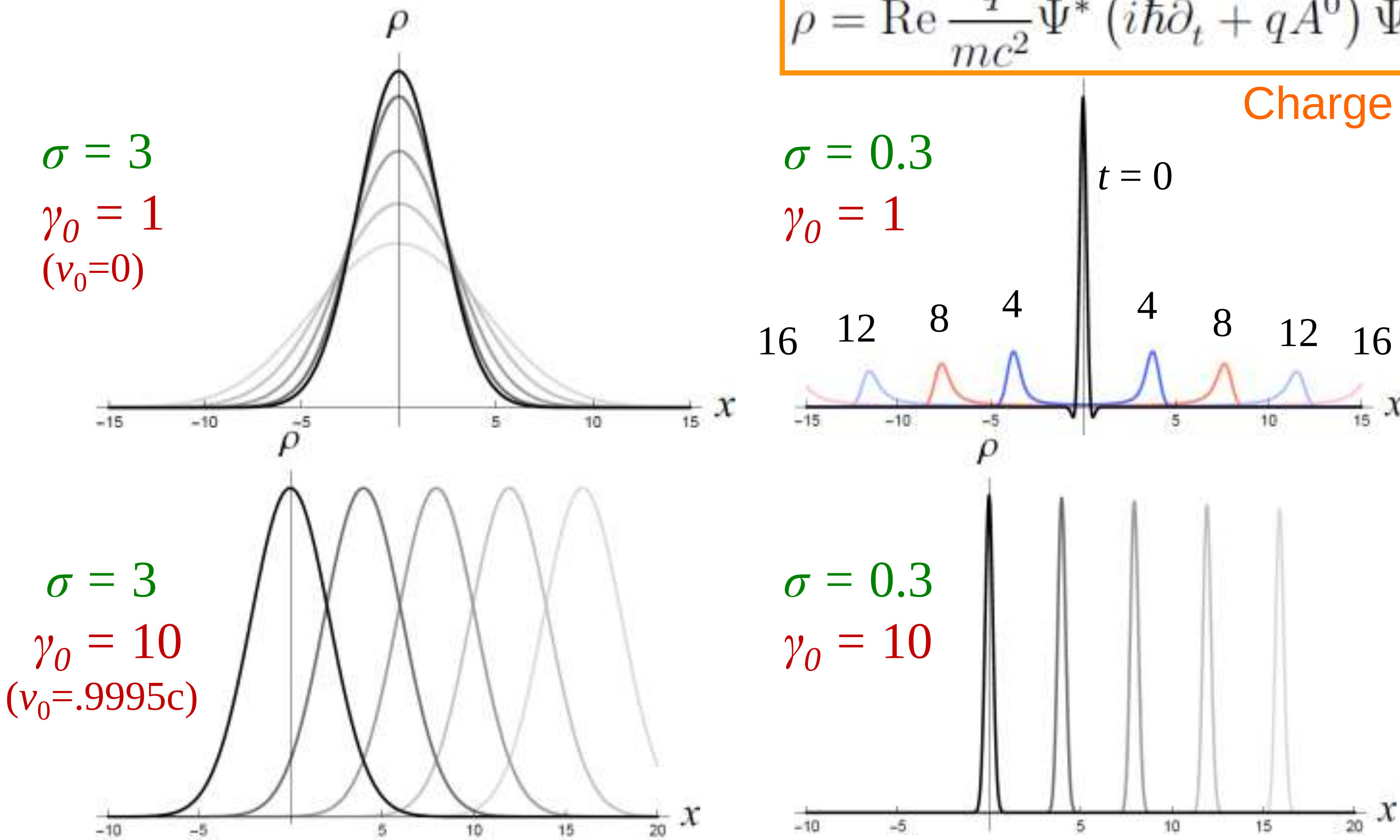
[Huang, He & SYL, Chinese J Phys 87(2024)486]

- Gaussian wavepackets of single "electron" – Solutions to Klein-Gordon eq.

$$\Psi(t,x) = \frac{\sqrt{\sigma}}{\sqrt{2\pi^{\frac{3}{2}}}} \int dp\, e^{-i[\omega(p)t - p(x-x_0)] - \frac{\sigma^2}{2}(p-p_0)^2}$$

$$\rho = \mathrm{Re}\,\frac{q}{mc^2}\Psi^*\left(i\hbar\partial_t + qA^0\right)\Psi$$

Charge density



Gaussian approx: Minimal initial width σ ~ (Compton WL λ_C) / (Lorentz factor γ_0)

Divergences and Regularization

- The *F-part* correlators often diverge in the coincidence limit, *e.g.*,

$$\langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle_F \equiv \lim_{t'\to t} \sum_{\mathbf{k},\mathbf{k}'} \frac{1}{2}\left(\mathcal{Z}^j_{(\lambda)\mathbf{k}}(t)\mathcal{Z}^{j'*}_{(\lambda')\mathbf{k}'}(t') + \mathcal{Z}^{j'}_{(\lambda)\mathbf{k}}(t')\mathcal{Z}^{j*}_{(\lambda')\mathbf{k}'}(t)\right) \langle 0|\hat{b}^{(\lambda)}_{\mathbf{k}}\hat{b}^{(\lambda')\dagger}_{\mathbf{k}'}|0\rangle$$

$$\propto \lim_{t',t'_0\to t,t_0} \int_{\tau(t_0)}^{\tau(t)+\epsilon_1} d\tilde{\tau} K(\tau(t),\tilde{\tau}) \int_{\tau(t'_0)+\epsilon_0}^{\tau(t')} d\tilde{\tau}' K(\tau(t')-\tilde{\tau}')$$

$$\int_0^\infty \frac{\omega^2 e^{-\omega\epsilon}}{(2\pi)^3 2\omega} d\omega \int d\Omega\, \mathcal{E}^j_{(\lambda)\mathbf{k}} \mathcal{E}^{(\lambda)j'}_{\mathbf{k}} \left[e^{-i\omega t+i\omega t'}(\cdots) + \cdots \right]$$

<u>Regulators:</u>

- Suppression of high-frequency contribution : $c\epsilon \sim \lambda_C / \gamma$

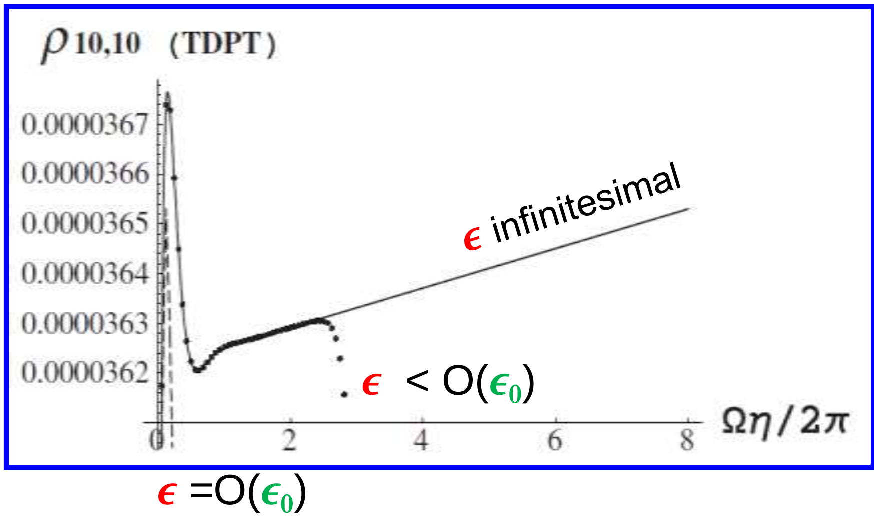
- Time resolution of the experiment / uncertainty of time tagging ϵ_0, ϵ_1

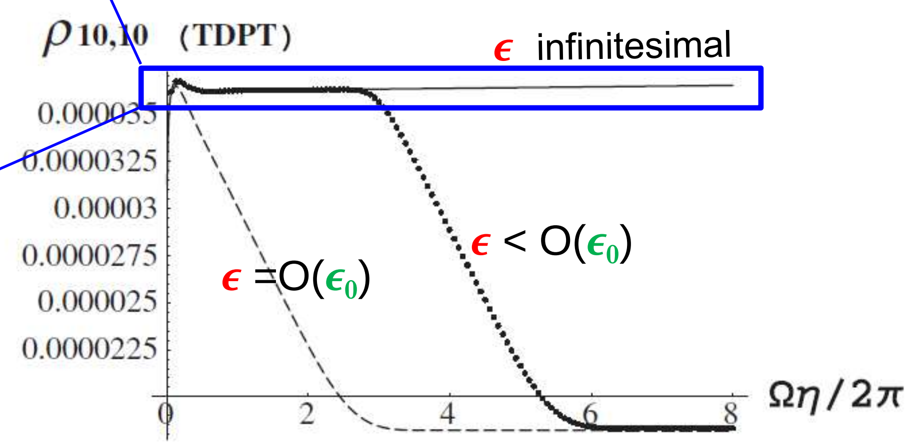
Planck scale?

Divergences and Regularization

- Not all the regulators are equal. For <u>uniformly accelerated</u> UD detectors, [SYL, BL Hu, PRD81(2010)045019]

If $\epsilon = O(\epsilon_0, \epsilon_1)$,
no Fermi golden rule (transition rate linear in t) can be seen.





- Time resolution of the observer
  "Physical cutoffs" ϵ_0, ϵ_1
- Suppression of high-frequency contribution
  "Mathematical cutoff" [infinitesimal in 2010, now we take $\epsilon \ll \epsilon_0, \epsilon_1$ in 2024]

Divergences and Regularization

- Transmission Electron Microscope in electron interference experiment [Tonomura et al, AmJPhys57(1989)117] (electrons have $1 < \gamma < 1.1$)

Source: Field-Emission (FE) Electron Gun

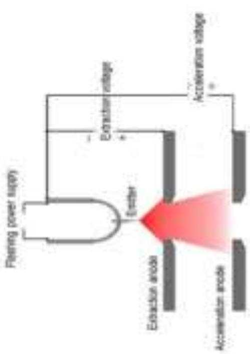
- Quantum tunneling
- Energy spread $\Delta E \approx 0.3$ eV

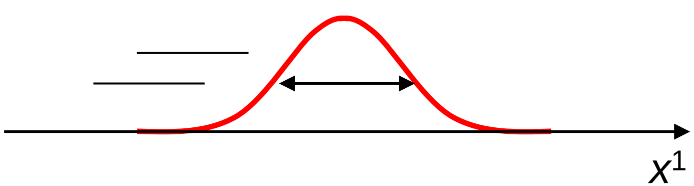
→ Temporal uncertainty

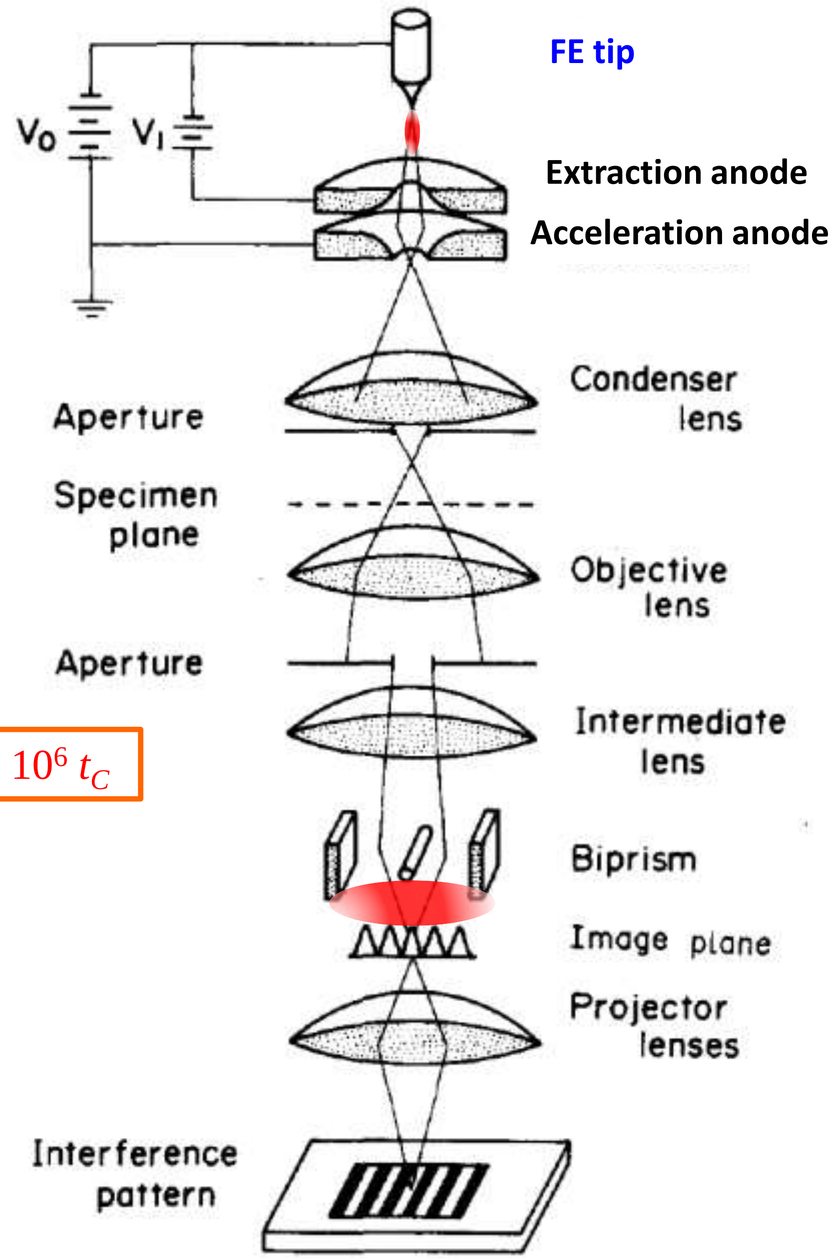
$$\Delta t \sim \hbar/\Delta E \approx 1.4 \text{ x } 10^{-14} \text{ s} \sim \epsilon_0 , \epsilon_1 \qquad \sim 10^6 \, t_C$$

→ Longitudinal coherent length

$$\Delta z^1 \sim v \, \Delta t \sim O(1 \ \mu\text{m})$$







https://www.jeol.com/words/semterms/20121024.021759.php#gsc.tab=0

Quantum coherence: Particle at rest

- Following [Tonomura et al 1989] ($1 < \gamma < 1.1$), we choose the initial widths of electron wavepackets as

1.7×10^{-6} m in longitudinal direction
5.0×10^{-9} m in transverse directions $\gg \lambda_C / \gamma$

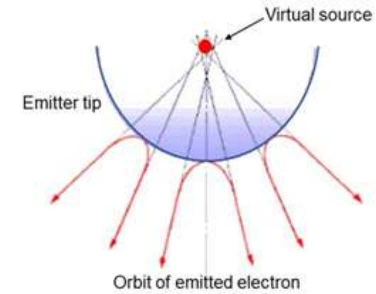
Assume $\mathbf{P} = 1$ at t = 0 [produced by tunneling].
As $t \uparrow$, the wavepacket spreads in all directions.

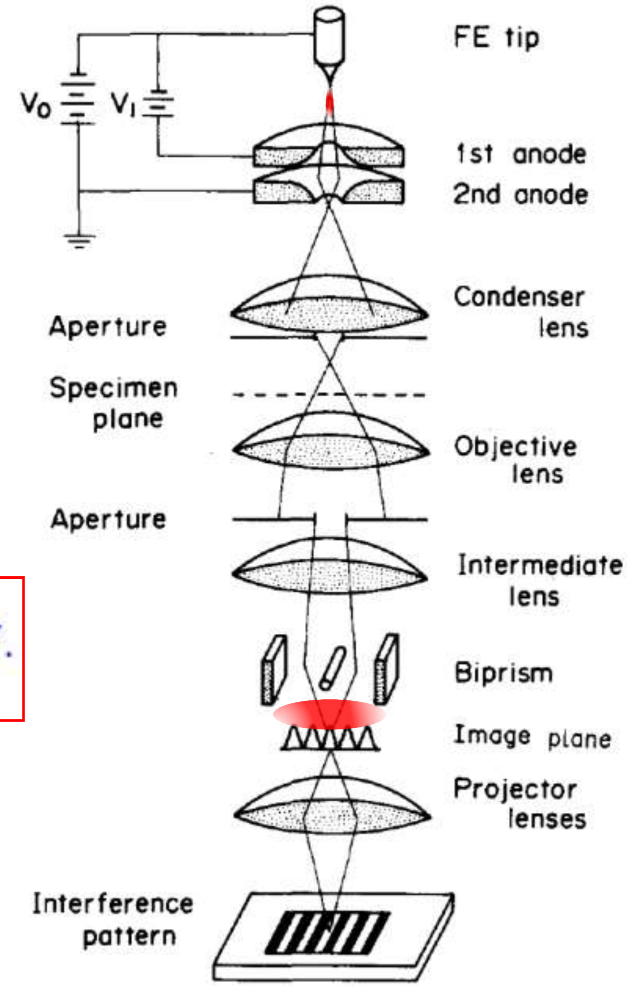
Around the flying time of the single electrons in the TEM, $t_F = 1.2 \times 10^{-8}$ s, we find

$$\mathbf{P}_1 \equiv \frac{\hbar}{2\sqrt{u_1}} \approx 0.9995 \qquad \mathbf{P}_\mathrm{T} \equiv \frac{\hbar}{2\sqrt{u_\mathrm{T}(t_F)}} \approx 0.47.$$

$$\mathbf{P} = \mathbf{P}_1 \mathbf{P}_2 \mathbf{P}_3 \approx 0.9995 \times (0.47)^2 \approx 0.22$$

[SYL, JPCS**2482**(2023)012018]
[SYL and Hu, JHEP 04(2024)065]





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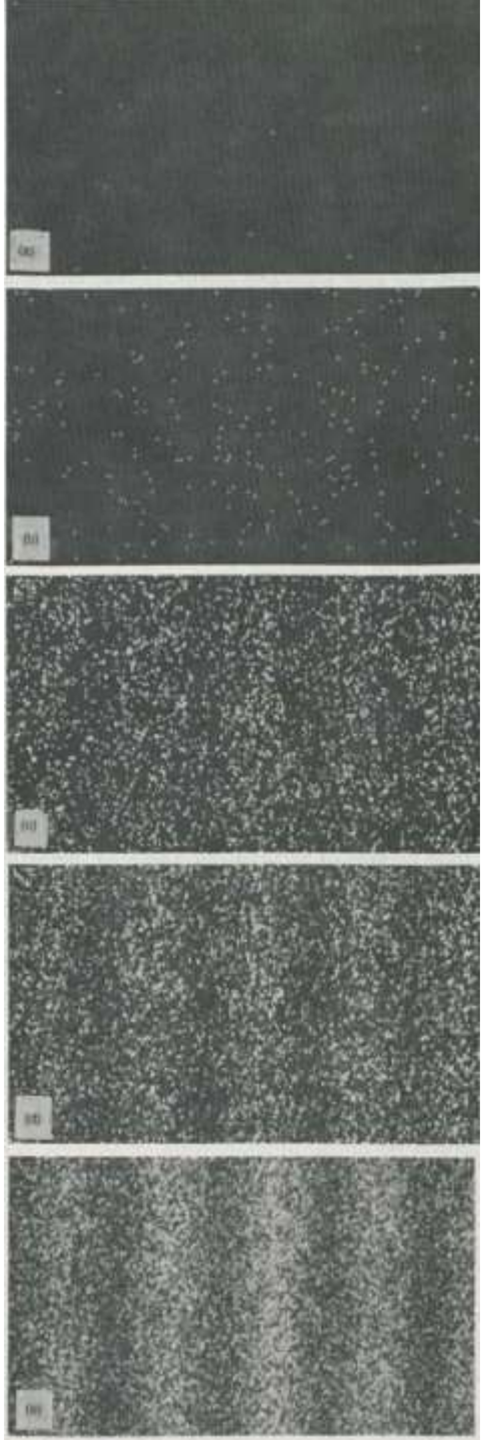
Field fluctuations may have blurred the
interference pattern in [Tonomura et al 1989].



Divergences and Regularization

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$$\langle \hat{z}^j(t), \hat{z}^{j'}(t)\rangle_F \equiv \lim_{t'\to t} \sum_{\mathbf{k},\mathbf{k}'} \frac{1}{2} \left(\mathcal{Z}^j_{(\lambda)\mathbf{k}}(t) \mathcal{Z}^{j'*}_{(\lambda')\mathbf{k}'}(t') + \mathcal{Z}^{j'}_{(\lambda)\mathbf{k}}(t') \mathcal{Z}^{j*}_{(\lambda')\mathbf{k}'}(t) \right) \langle 0| \hat{b}^{(\lambda)}_{\mathbf{k}} \hat{b}^{(\lambda')\dagger}_{\mathbf{k}'} |0\rangle$$

$$\propto \lim_{t',t'_0 \to t,t_0} \int_{\tau(t_0)}^{\tau(t)+\epsilon_1} d\tilde{\tau} K(\tau(t),\tilde{\tau}) \int_{\tau(t'_0)+\epsilon_0}^{\tau(t')} d\tilde{\tau}' K(\tau(t')-\tilde{\tau}')$$

$$\int_0^\infty \frac{\omega^2 e^{-\omega\epsilon}}{(2\pi)^3 2\omega} d\omega \int d\Omega\, \mathcal{E}^j_{(\lambda)\mathbf{k}} \mathcal{E}^{(\lambda)j'}_{\mathbf{k}} \left[e^{-i\omega t + i\omega t'} (\cdots) + \cdots \right]$$

Regulators:

- Suppression of high-frequency contribution : $c\,\epsilon \sim \lambda_C / \gamma(t)$

- Time resolution of the experiment / uncertainty of time tagging ϵ_0, ϵ_1

$$\epsilon_0 \approx \epsilon'_0/\bar{\gamma}[\tau(t_0)] \qquad \epsilon_1 \approx \epsilon'_1/\bar{\gamma}[\tau(t)]$$

… **can be time-dependent for accelerated particles in the laboratory frame.**

IV. Unruh effect

Charged particle in a uniform electric field

- In $\bar{F}^{01}_{[0]} = -\bar{F}^{10}_{[0]} = \mathcal{E}/c$,

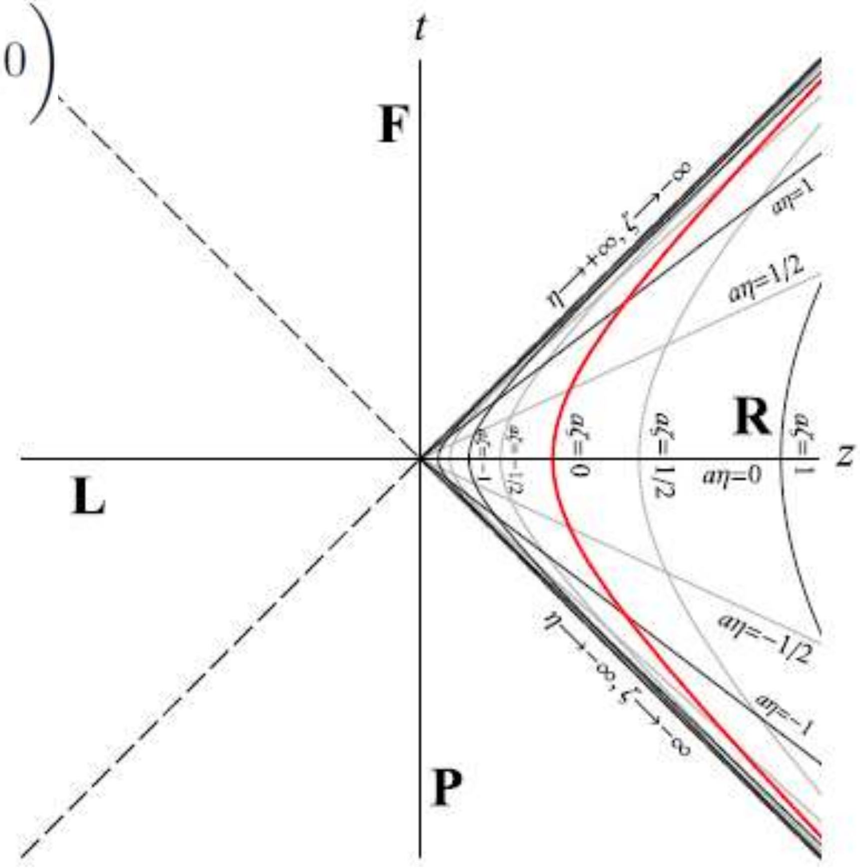
  a charged particle moves along the classical trajectory

$$\bar{z}^{\mu}(\tau) = \left(\frac{c^2}{a} \sinh \frac{a\tau}{c}, \frac{c^2}{a} \cosh \frac{a\tau}{c}, 0, 0 \right)$$

(uniformly accelerated charge)

with $a = q\mathcal{E}/\bar{m}$.

Let $\alpha = a/c$.



Charged particle in a uniform electric field

- Longitudinal deviation (parallel to the direction of acceleration)

$$\bar{m}\partial_\tau\left[\bar{\gamma}^2(\tau)\partial_\tau \mathcal{Z}_1^\Omega(\tau)\right] = qc\bar{\gamma}(\tau)\left(1+s\partial_\tau\right)\mathcal{F}_{10}^{[0]\Omega}\left(\bar{z}(\tau)\right) + O(\Lambda^{-1})$$

Sol: $\bar{m}\mathcal{Z}_1^\Omega(\tau) = C_1^\Omega + \tilde{C}_1^\Omega \tanh\alpha\tau + qc\int_{\tau_0}^{\tau} d\tilde{\tau} K_\parallel(\tau,\tilde{\tau})\,\bar{\gamma}(\tilde{\tau})\left(1+s\partial_{\tilde{\tau}}\right)\mathcal{F}_{10}^{[0]\Omega}\left(\bar{z}(\tilde{\tau})\right)$

with the kernel $K_\parallel(\tau,\tau') \equiv \alpha^{-1}\left(\tanh\alpha\tau - \tanh\alpha\tau'\right)$

- Transverse deviations (perpendicular to the acceleration)

$$\bar{m}\left[\left(1+s^2\alpha^2\right)\partial_\tau^2 + s\alpha^2\partial_\tau\right]\mathcal{Z}_\top^\Omega = qc(1+s\partial_\tau)\mathcal{F}_\top^\Omega + O(\Lambda^{-1})$$

where $\mathcal{F}_\top^\Omega(\tau) \equiv \mathcal{F}_{\top 0}^{[0]\Omega}\left(\bar{z}(\tau)\right)\cosh\alpha\tau + \mathcal{F}_{\top 1}^{[0]\Omega}\left(\bar{z}(\tau)\right)\sinh\alpha\tau.$

Sol: $\bar{m}\mathcal{Z}_\top^\Omega(\tau) = C_\top^\Omega + \tilde{C}_\top^\Omega e^{-\frac{s\alpha^2}{\varsigma}(\tau-\tau_0)} + qc\int_{\tau_0}^{\tau} d\tilde{\tau} K_\perp(\tau,\tilde{\tau})\left(1+s\partial_{\tilde{\tau}}\right)\mathcal{F}_\top^\Omega(\tilde{\tau})$

with $\varsigma \equiv 1+(s\alpha)^2$ and the kernel $K_\perp(\tau,\tilde{\tau}) \equiv \frac{1}{s\alpha^2}\left(1-e^{-\frac{s\alpha^2}{\varsigma}(\tau-\tilde{\tau})}\right)$

Unruh effect: Not a straightforward story

- Two-point correlator of particle position deviation in the direction of *E* field

$$\langle \hat{z}^1(t), \hat{z}^1(t')\rangle_F = \frac{\hbar}{(2\pi)^3\varepsilon_0} \times$$

$$\mathrm{Re}\int_0^\infty \frac{\omega^2}{c^2}\frac{d\omega}{2\omega c}\int_0^{2\pi} d\varphi \int_0^\pi d\theta\, \sin\theta \frac{qc}{\bar{m}}\mathcal{E}_{\mathbf{k}}^{(\lambda)j}{}_0 \frac{qc}{\bar{m}}\mathcal{E}^{\mathbf{k}\ j'*}_{(\lambda)\ 0} f_{\mathbf{k}}^{\parallel}(\tau) f_{\mathbf{k}}^{\parallel *}(\tau')$$

where $f_{\mathbf{k}}^{\parallel}(\tau) = \int_{\tau_0}^{\tau} d\tilde{\tau} K_{\parallel}(\tau,\tilde{\tau})\bar{\gamma}(\tilde{\tau})\left[1 + s\, ik_\nu \dot{\bar{z}}^\nu(\tilde{\tau})\right] e^{ik_\mu \bar{z}^\mu(\tilde{\tau})} e^{-\omega\epsilon/2}$

First, compute the integrals over the *k*-space, i.e.,

$$\mathcal{I}_{BB'}^{jj'}(\tilde{\tau},\tilde{\tau}') \equiv \int \omega d\omega d\varphi\, d\cos\theta\, \mathcal{E}_{\mathbf{k}}^{(\lambda)j}{}_B\, \mathcal{E}^{\mathbf{k}\ j'*}_{(\lambda)\ B'}\, e^{-i\frac{\omega}{c}\left[\bar{z}^0(\tilde{\tau}) - \bar{z}^0(\tilde{\tau}')\right] + ik_1\left[\bar{z}^1(\tilde{\tau}) - \bar{z}^1(\tilde{\tau}')\right] - \omega\epsilon}$$

Then we find

$$\mathcal{I}_{00}^{11}(\tilde{\tau},\tilde{\tau}') = 8c^2 \int \frac{d\bar{\kappa} e^{-i\bar{\kappa}(\tilde{\tau}-\tilde{\tau}')} d\check{\Delta} e^{i\bar{\kappa}\check{\Delta}}}{\left[\frac{4c^2}{\alpha^2}\sinh^2\frac{\alpha\check{\Delta}}{2} - i\epsilon c\left(\frac{4c}{\alpha}\cosh\alpha\tilde{T}\sinh\frac{\alpha\check{\Delta}}{2} - i\epsilon c\right)\right]^2}$$

$$\tilde{T} \equiv (\tilde{\tau} + \tilde{\tau}')/2$$

Unruh effect: Not a straightforward story

- Two-point correlator of particle position deviation in the direction of *E* field

$$\mathcal{I}^{11}_{00}(\tilde{\tau},\tilde{\tau}') = 8c^2 \int \frac{d\bar{\kappa} e^{-i\bar{\kappa}(\tilde{\tau}-\tilde{\tau}')} d\check{\Delta} e^{i\bar{\kappa}\check{\Delta}}}{\left[\frac{4c^2}{\alpha^2}\sinh^2\frac{\alpha\check{\Delta}}{2} - i\epsilon c\left(\frac{4c}{\alpha}\cosh\alpha\tilde{T}\sinh\frac{\alpha\check{\Delta}}{2} - i\epsilon c\right)\right]^2}$$

$\tilde{T} \equiv (\tilde{\tau}+\tilde{\tau}')/2$

However, textbooks [e.g. Birrell and Davies] say that we should do the following integral to get the Unruh effect,

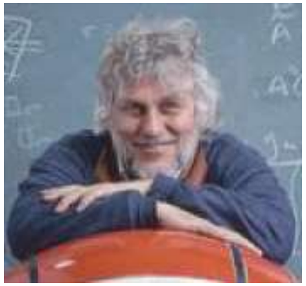
$$\mathcal{I}^{11}_{00}(\tilde{\tau},\tilde{\tau}') \approx 8c^2 \int d\bar{\kappa} e^{-i\bar{\kappa}(\tilde{\tau}-\tilde{\tau}')} \int d\check{\Delta} \frac{e^{i\bar{\kappa}\check{\Delta}}}{\left[\frac{4c^2}{\alpha^2}\sinh^2\frac{\alpha}{2}(\check{\Delta}-i\epsilon')\right]^2}$$

These two are not equivalent, unless 1) we set the regulator

$c\epsilon \sim \lambda_C/\bar{\gamma}(\tau(t))$: length contraction,

and 2) $\bar{\gamma}(\tau_0)$ (or initial speed) is not too large in the lab frame.

Unruh effect: Not a straightforward story



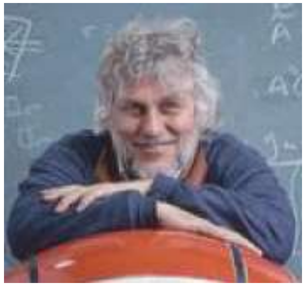
Then we have

$$\mathcal{I}_{00}^{11}(\tilde{\tau},\tilde{\tau}') = \frac{8\pi}{3c^2}\int d\kappa e^{-i\kappa(\tilde{\tau}-\tilde{\tau}')}\frac{\kappa(\kappa^2+\alpha^2)}{1-e^{-2\pi\kappa/\alpha}}$$

~ in a bosonic bath at the Unruh temperature $T_U = \dfrac{\hbar a}{2\pi k_B c}$

See Unruh effect on 1) spreading of wavepacket in position & momentum spaces, and so 2) decoherence.

Unruh effect: Not a straightforward story



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~ in a bosonic bath at the Unruh temperature $T_U = \frac{\hbar a}{2\pi k_B c}$

See Unruh effect on 1) spreading of wavepacket in position & momentum spaces, and so 2) decoherence.

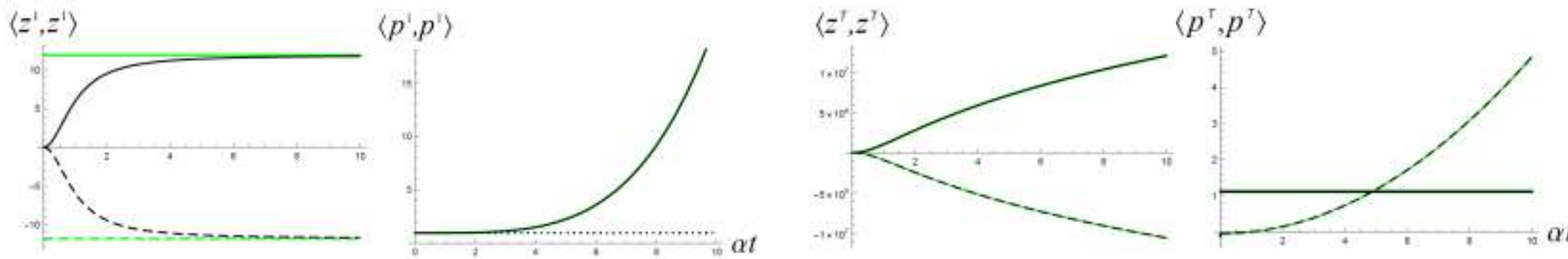
- In the longitudinal momentum correlator $\langle \hat{p}^1(t), \hat{p}^1(t')\rangle$, we have a few terms proportional to

$$-i\pi\cosh\alpha\tilde{T}\int d\kappa\, e^{-i\kappa(\tilde{\tau}-\tilde{\tau}')-\kappa\epsilon'}\frac{4\kappa^2+\alpha^2}{1+e^{-2\pi\kappa/\alpha}}$$

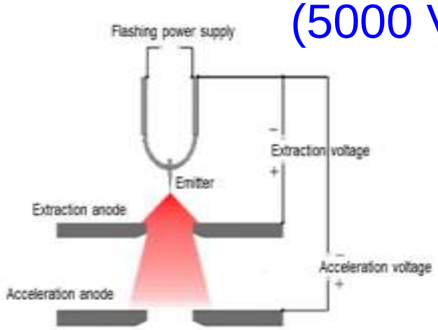
~ in a *fermionic* bath at the Unruh temperature?!

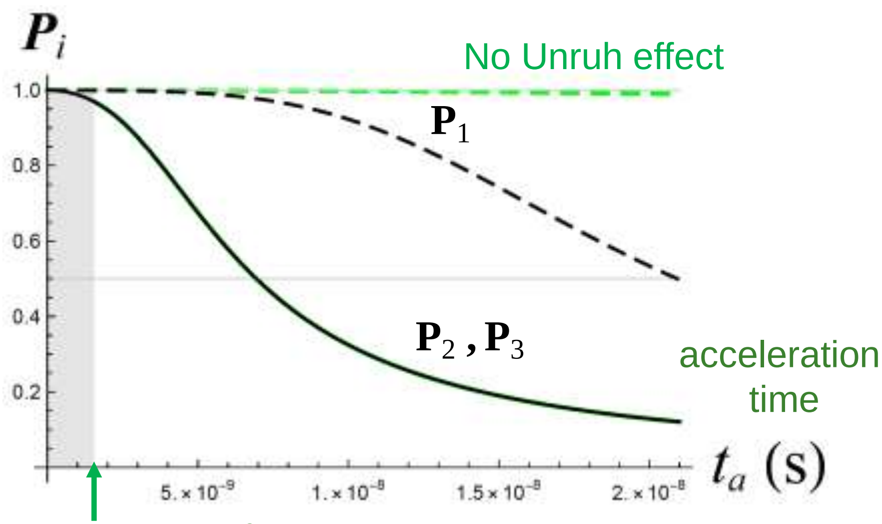
Quantum coherence: Uniformly accelerated charge

- Still choose the initial widths of electron wavepackets as
  1.7 x 10^{-6} m in longitudinal direction, 5.0 x 10^{-9} m in transverse directions



acceleration produced
by electric field of 5 x 10^5 N/C
(5000 V/cm)





1.54 x 10^{-9} s in Tomomura's microscope

Electron coherence is indeed not reduced seriously
during the acceleration stage in [Tonomura et al 1989].

Summary [SYL, BL Hu, JHEP 04(2024)065]

1. We have developed a linearized effective theory of single spinless charged particles in EM fields.
2. We have got more knowledge on the divergences and regulators. The values of the regulators are chosen according to experiments, and will be time-dependent in the lab frame if the particle speed is.
3. Evolution of particle correlators and quantum coherence of flying single electrons have been obtained. Field fluctuations may be the major source of blurring the interference pattern in Tonomura's experiments.
4. The Unruh effect on the charged particle in uniform acceleration has been identified. It is more significant in decoherence of longitudinal deviations.

On-going projects

- Quantum radiation emitted by an accelerated charge.
- Quantum information processes using flying single electrons (particularly in electron microscopes), toy model for gravitational interactions between flying masses (the BMV problem).

Thank You!