Calculating electromagnetic properties of the quantum vacuum using electrodynamics to describe the interaction of a photon with vacuum fluctuations

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The International Association for Relativistic Dynamics 14th Biennial Conference
Aalto University, Espoo, Finland
3-6 June 2024

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Today	$2.99 imes 10^{11} \mathrm{m}$	16.7 min	$3.00 imes 10^8 \text{ m/s}$

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- ➤ To calculate the speed *c* of light in the vacuum, the physical system is a photon traveling through the quantum vacuum.
- ► The speed c of light in the vacuum is an electromagnetic property of the quantum vacuum. Thus it is possible to calculate c using Maxwell's equations, quantum electrodynamics, and quantum mechanics to describe the interaction of a photon with the quantum vacuum.

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- ► Vacuum energy is the energy source for the creation of type 1 vacuum fluctuations.

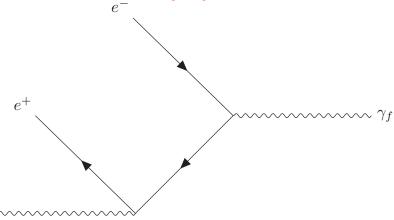
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- appear as external particles in Feynman diagrams.

FEYNMAN DIAGRAM OF A TYPE 1 ELECTRON-POSITRON VACUUM FLUCTUATION INTERACTING WITH A PHOTON



cannot exert a force on ordinary matter.

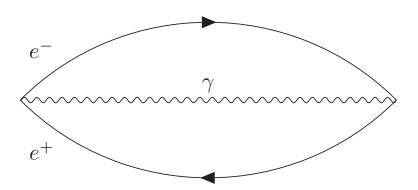
- cannot exert a force on ordinary matter.
- ▶ If they could exert a force, they could do work on ordinary particles. Then when a vacuum fluctuation vanished, the energy associated with any work done by the vacuum fluctuation would remain. permanently violating conservation of ordinary energy.

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- ▶ are off shell: $(mc^2)^2 \neq E^2 (\mathbf{P}c)^2$

FEYNMAN DIAGRAM FOR A TYPE 2 VACUUM FLUCTUATION: (VACUUM DIAGRAM OR VACUUM BUBBLE)



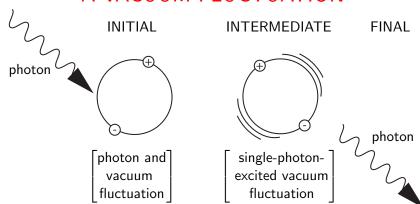
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- ▶ While present, type 1 vacuum fluctuations slow the progress of photons, similarly to the way that an ordinary dielectric slows photons.
- ➤ The slowing of photons by vacuum fluctuations determines the permittivity of the vacuum, the speed of light in the vacuum, and the fine-structure constant.

PHOTON POLARIZING A VACUUM FLUCTUATION



EARLY LITERATURE DISCUSSING VACUUM FLUCTUATIONS

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- ▶ Pauli & Weisskopf (1934) discussed the idea that the vacuum could be treated as a medium with electric and magnetic polarizability.

RESULTS: THEORETICAL FORMULA FOR ϵ_0

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- ► c is calculated from properties of the quantum vacuum, Maxwell's equations, QED, and quantum mechanics so the calculated value of c is automatically the same in all directions in every inertial reference frame.

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Charged-lepton-antilepton vacuum fluctuations contribute much more to the permittivity of the vacuum than quark-antiquark vacuum fluctuations.

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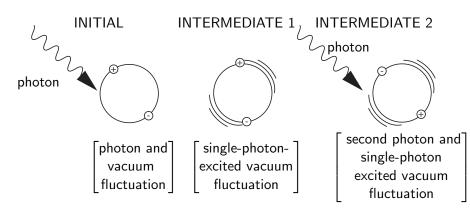
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- Number density $_{\tau-\bar{\tau}~\mathrm{VF}}=4.7\times10^{49}/\mathrm{m}^3$
- For comparison, the number density of atoms or molecules in an ideal gas at STP= $2.7 \times 10^{25}/\text{m}^3$

TWO PHOTONS POLARIZING A VACUUM FLUCTUATION



TWO PHOTONS POLARIZING A VACUUM FLUCTUATION - CONTINUED

INTERMEDIATE 3

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FINAL

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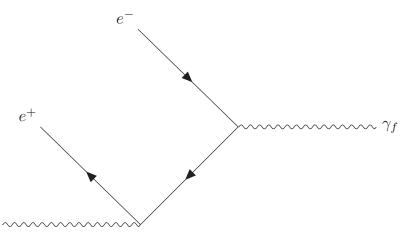
two-photonexcited vacuum fluctuation

2 photons

WHY OUR RESULTS MAY BE CORRECT

▶ The values for ϵ_0 , c, and α are correct to within a few percent.

FEYNMAN DIAGRAM OF A PHOTON-EXCITED PARAPOSITRONIUM VACUUM FLUCTUATION DECAYING INTO A PHOTON



WHY OUR RESULTS MAY BE CORRECT

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- ▶ The values for ϵ_0 , c, and α are all correct to within a few percent.
- ► The calculated value of *c* automatically satisfies the second postulate of special relativity, eliminating the need for the postulate.
- The electromagnetic properties of the quantum vacuum are calculated using standard physics (Maxwell's equations, quantum electrodynamics, and quantum mechanics) to describe the interaction of a photon with the quantum vacuum. The only new physics is interaction of photons with the quantum vacuum.

RESPONSE FROM THE PHYSICS COMMUNITY

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- ► Fewer than 10 people have contacted us: to date no one has pointed out an error.

ONGOING RESEARCH

 Calculation of electromagnetic properties of the vacuum to next order in the fine-structure constant α.

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- Calculation of electromagnetic properties of the vacuum to next order in the fine-structure constant α.
- Calculation of the effect of a vacuum fluctuation interacting with two photons.

ONGOING RESEARCH

- ▶ Calculation of electromagnetic properties of the vacuum to next order in the fine-structure constant α .
- Calculation of the effect of a vacuum fluctuation interacting with two photons.
- ► Calculation of the gravitational constant *G*, a gravitational property of the quantum vacuum.